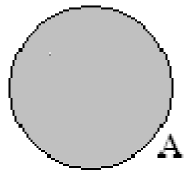
Section 11 - Introduction to Set Operations

Venn Diagrams

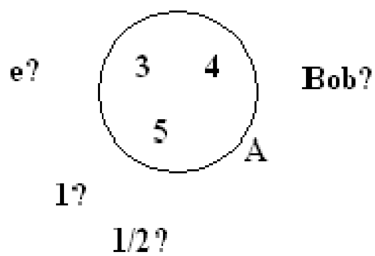
Suppose we have a set A. Some objects are in A and some objects are not. We could represent this as follows:



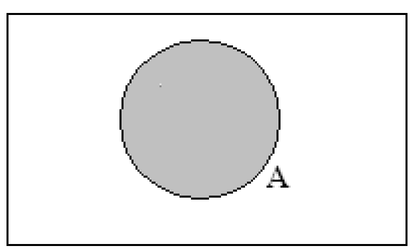
The Set A

Then, anything in A could be thought of as being inside the circle and anything not in A could be thought of as outside the circle.

There is a slight problem wiht this, however. Suppose, for example, that $A = \{3, 4, 5\}$. Then, it's easy to put the correct elements in A, but what isn't in A?



For this reason, we usually draw a rectangle on the outside of our circle (representing the Universal Set) to mean all of the objects we are considering. So, if I am only thinking about Integers, then the elements living in the rectangle, but not inside the circle, would be the integers that are not in A.



The Set A

Now, suppose we have two sets, A and B.

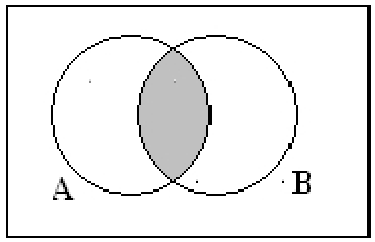
There are several ways we can consider the two sets. We can think about what elements they have in common (if any), what the combined set would look like, what elements appear on one of the lists but not the other, what elements are on neither, etc.

Set Intersection

Def: Let A and B be sets. Then the intersection of A and B, denoted by $A \cap B$ is the set of all elements that are in both A and B.

Ex: If $A = \{3, 6, 9\}$ and $B = \{1, 2, 3, 4, 5, 6\}$, then $A \cap B = \{3, 6\}$.

Using Venn Diagrams, we have:



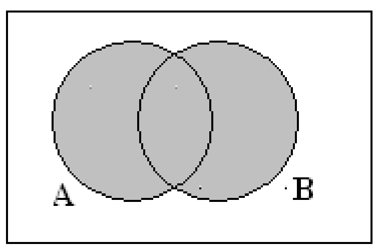
The set $A \cap B$

Set Union

Def: Let A and B be sets. Then the union of A and B, denoted by $A \cup B$ is the set of all elements that are in A or B.

Ex: If $A = \{3, 6, 9\}$ and $B = \{1, 2, 3, 4, 5, 6\}$, then $A \cup B = \{1, 2, 3, 4, 5, 6, 9\}$.

Using Venn Diagrams, we have:



The set $A \cup B$

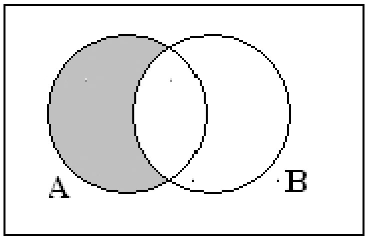
Note: For all sets A and B, $|A \cup B| \geq |A \cap B|$.

Set Difference

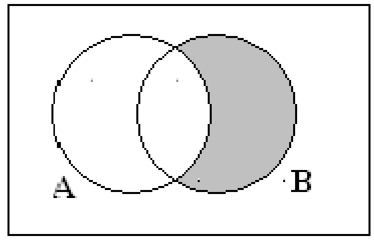
Def: Let A and B be sets. Then the set difference $A - B$ is the set of all elements that are in A, but not B.

Ex: If $A = \{3, 6, 9\}$ and $B = \{1, 2, 3, 4, 5, 6\}$, then $A - B = \{9\}$ and $B - A = \{1, 2, 4, 5\}$

Using Venn Diagrams, we have:



The Set A - B



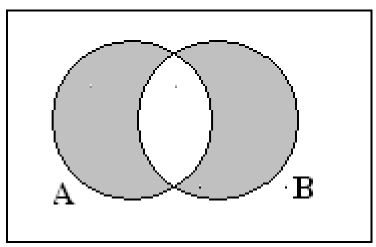
The Set B - A

Symmetric Difference

Def: Let A and B be sets. Then the symmetric difference $A \triangle B$ is the set of all elements that are in A or B but not both.

Ex: If $A = \{3, 6, 9\}$ and $B = \{1, 2, 3, 4, 5, 6\}$, then $A \triangle B = \{1, 2, 4, 5, 9\}$.

Using Venn Diagrams, we have:



The set $A \triangle B$

Note: For all sets A and B, $A \triangle B = (A - B) \cup (B - A)$.

Cartesian Product

This one is not like the others. When we take the cartesian product of two sets, we do not create a new set with elements similar to those in A and B. We create a new set with elements made up of two-element lists (ordered pairs) of elements similar to those in A and B.

Def: Let A and B be sets. Then the Cartesian Product $A \times B = \{(a, b) : a \in A, b \in B\}$.

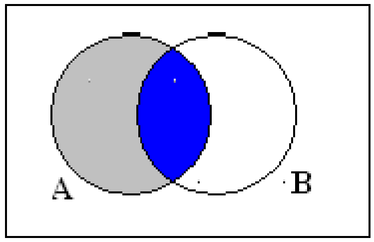
Ex: If $A = \{3, 6, 9\}$ and $B = \{1, 2, 3, 4, 5, 6\}$, then $A \times B = \{(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (6, 1), (6, 2$

There is no useful Venn Diagram for this.

Note: For all sets A and B, $|A \times B| = |A| \cdot |B|$.

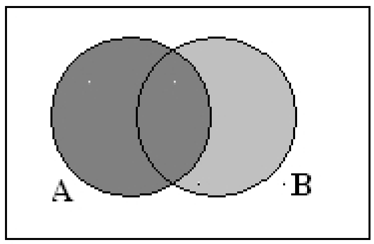
Combining Operations.

Ex: Sketch a Venn Diagram illustrating $(A - B) \cup (A \cap B)$



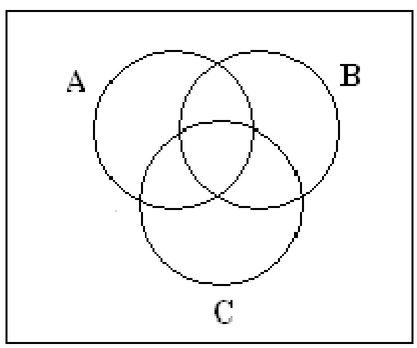
Note that the above gives an illustrative argument that $(A - B) \cup (A \cap B) = A$. Is it a proof? Maybe. But, not for the purposes of this class.

Ex: Give a Venn Diagram illustrating $(A \cup B) \cap A = A$.

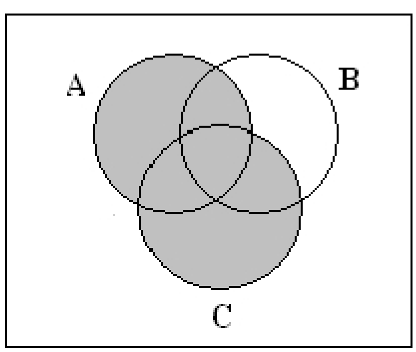


Working with three sets at a time

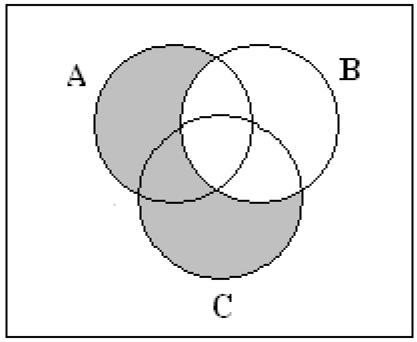
Suppose we have a third set, C, we want involved. First, we need to figure out how it should enter our diagram. We need to be careful that we are leaving open all possibilities.



Now, let's illustrate $(A \cup C)$ and $(A \cup C) - B$.

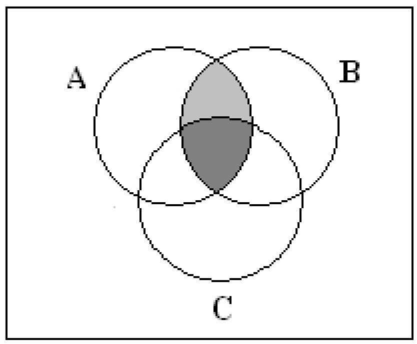


$A \cup C$

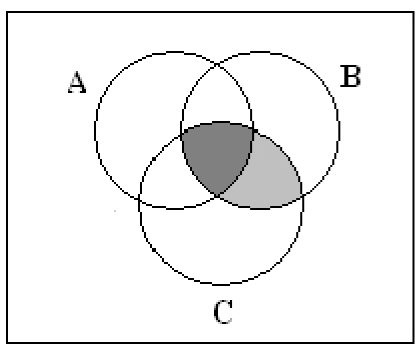


$(A \cup C) - B$

Ex: Give a Venn Diagram illustrating $(A \cap B) \cap C = A \cap (B \cap C)$.



$(A \cap B) \cap C$



$A \cap (B \cap C)$

Homework: Section 11, P. 74 #1,3,23