Section 11 cont'd - Proving theorems about set operations.

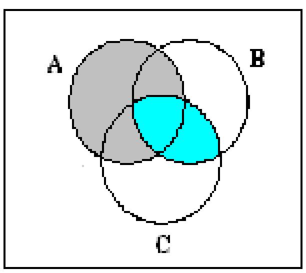
To prove statements about $A \cup B, A \cap B, A - B$, etc. The following table is useful:

Statement:	Implies/Is implied by:
$x \in A \cup B$	$x \in A$ or $x \in B$
$x \in A \cap B$	$x \in A$ and $x \in B$
$x \in A - B$	$x \in A$ and $x \notin B$
$x \in A \triangle B$	Either $x \in A$ and $x \notin B$ or $x \in B$ and $x \notin A$
$x \in A \times B$	$x = (a, b)$ for some $a \in A, b \in B$.

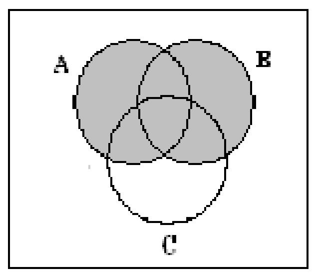
Additionally : $x \in A$ implies $x \in A \cup B$ for any set B

and, $x \in \emptyset$ is a contradiction.

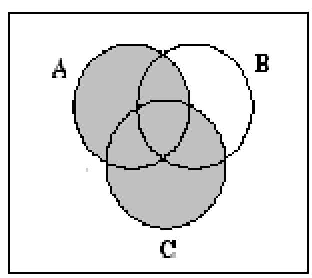
Ex: Illustrate: $\forall$ sets A, B, C, $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.
Ans:



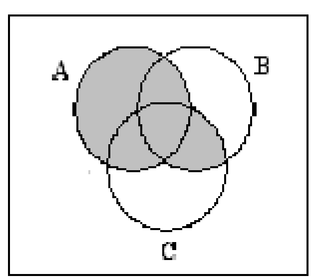
$A \cup (B \cap C)$



$(A \cup B)$



$(A \cup C)$



$(A \cup B) \cap (A \cup C)$

Ex: Prove: $\forall$ sets A, B, C, $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.
Ans (Method 1):

Proof:
($\subseteq$) Let $x \in A \cup (B \cap C)$
Then $x \in A$ or $x \in (B \cap C)$.
That is, either $x \in A$ or $x \in B$ and $x \in C$.
If $x \in A$ then $x \in A \cup B$ and $x \in A \cup C$.
On the other hand, if $x \in B$ and $x \in C$, then $x \in A \cup B$ and $x \in A \cup C$.
Thus, in either case, $x \in (A \cup B) \cap (A \cup C)$.
Therefore, $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$.

Now, have the class do the opposite direction.

Ans: (Method 2):

$$\begin{aligned}
A \cup (B \cap C) &= \{x : x \in A \cup (B \cap C)\} \\
&= \{x : x \in A \ \vee \ x \in (B \cap C)\} \\
&= \{x : x \in A \ \vee (\ x \in B \ \wedge \ x \in C)\} \\
&= \{x : (x \in A \ \vee \ x \in B) \ \wedge \ (x \in A \vee x \in C)\} \\
&= \{x : (x \in A \cup B) \ \wedge \ (x \in A \cup C)\} \\
&= \{x : x \in (A \cup B) \cap (A \cup C)\} \\
&= (A \cup B) \cap (A \cup C)
\end{aligned}$$

Now do worksheet.

Homework: Section 11, P. 74 #7,10-12,16,17,21,22,25