

Section 13 - Relations

Consider the set  $A = \{5, 6, 7\}$  and  $B = \{2, 3\}$ .

Then  $A \times B = \{(5, 2), (5, 3), (6, 2), (6, 3), (7, 2), (7, 3)\}$  and  $A \times A = \{(5, 5), (5, 6), (5, 7), (6, 5), (6, 6), (6, 7), (7, 5), (7, 6), (7, 7)\}$ .

These are sets whose elements are ordered pairs. So,  $(5, 3) \in A \times B$  and  $(7, 2) \in A \times B$ , and so on. Now, if we create a subset of  $A \times B$  or of  $A \times A$ , it will be a set whose elements are ordered pairs, also.

So, for example,  $\{(5, 3), (7, 2)\} \subseteq A \times B$  and  $\{(5, 5), (5, 6), (6, 5), (7, 5), (7, 6)\} \subseteq A \times A$ .

Def: Let  $A$  be a set. A relation,  $R$ , on  $A$  is a subset of  $A \times A$ .

Def: Let  $A$  and  $B$  be sets. A relation,  $R$ , from  $A$  to  $B$  is a subset of  $A \times B$ .

Some notation:  $aRb$  means  $(a, b) \in R$ .  $R^{-1}$  is the inverse relation of  $R$  and is formed by reversing the order of each element of  $R$ . Note, if  $R$  is a relation from  $A$  to  $B$  then  $R^{-1}$  is a relation from  $B$  to  $A$ .

Ex: Let  $A = \{0, 5, 10, 15\}$  and  $B = \{0, 2, 4, 6, 8, 10\}$ . Let  $R$  be the relation  $\{(0, 0), (0, 2), (0, 4), (0, 6), (0, 8), (2, 0), (4, 0), (6, 0), (8, 0), (10, 0)\}$ .

1. Find  $R^{-1}$ .

$$R^{-1} = \{(0, 0), (2, 0), (4, 0), (6, 0), (8, 0), (10, 2), (10, 10)\}$$

2. True or False?

(a)  ${}_0R_2$  (T)

(b)  ${}_2R_0$  (F)

(c)  ${}_2R_0^{-1}$  (T)

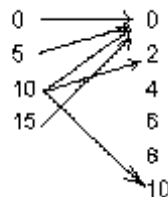
(d)  ${}_{10}R_{10}$  (T)

(e)  ${}_{10}R_{10}^{-1}$  (T)

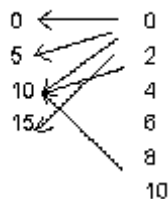
3. Note that  $R$  is made up of all the possible ordered pairs such that the first is a multiple of the second. Write  $R$  using set notation.

$$R = \{(x, y) : x \in A, y \in B, y \mid x\}$$

4. Represent  $R$  with a diagram.



5. Represent  $R^{-1}$  with a diagram.



Ex: Let  $R$  be the relation on  $\mathbb{Z}$  given by  $R = \{(x, y) : x, y \in \mathbb{Z}, y = x + 1\}$ .

1. Find  $R^{-1}$ .

$$R^{-1} = \{(x, y) : x, y \in \mathbb{Z}, x = y + 1\}$$

OR

$$R^{-1} = \{(y, x) : x, y \in \mathbb{Z}, y = x + 1\}$$

2. True or False?

(a)  $(5, 9) \in R$ . (F)

(b)  ${}_5R_9$ . (F)

(c)  $(2, 3) \in R$ . (T)

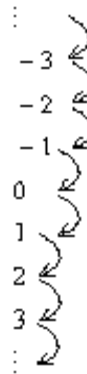
(d)  ${}_2R_3$ . (T)

(e)  ${}_{-1}R_0$ . (T)

(f)  ${}_{-1}R_0^{-1}$  (F)

(g)  ${}_0R_{-1}^{-1}$  (T)

3. Represent  $R$  with a diagram.



4. Represent  $R^{-1}$  with a diagram.

