

## Section 14 - Equivalence Relations and Congruence Modulo $n$

Def: Let  $R$  be a relation on a set  $A$ . If  $R$  is reflexive, symmetric, and transitive, then  $R$  is called an equivalence relation.

Ex: Let  $A = \{a, b, c, d\}$ , let  $R = \{(a, a), (b, b), (c, c), (d, d), (b, c), (c, b)\}$  and  $S = \{(a, a), (b, b), (c, c), (d, d)\}$ . Both are ERs.

Ex: Let  $R$  be the a relation on  $\mathbb{Z}$  given by  $xR_y$  if  $x = y$ . This is an ER.

Ex: Let  $A$  be you guys. Let  $R$  be given by  $xR_y$  if  $x$  and  $y$  are the same height, rounded to the nearest inch. Then  $R$  is an ER.

Ex: Let  $R$  be the relation on  $\mathbb{Z}$  given by  $xR_y$  if  $x^2 \mid y^2$ . This is reflexive and transitive, but not symmetric. So  $R$  is not an ER.

Ex: Let  $R$  be the relation on  $\mathbb{Z}$  given by  $xR_y$  if  $3 \mid (x - y)$ . This is an ER (you will prove this on the worksheet in a moment.)

Some notation: If  $a \mid (x - y)$ , we write  $y \equiv x \pmod{a}$ . Thus, the relation above could be written as  $xR_y$  if  $x \equiv y \pmod{3}$  (or as  $y \equiv x \pmod{3}$ ). We say ' $x$  and  $y$  are congruent modulo 3'.or ' $x$  and  $y$  are equivalent modulo 3'

Now do worksheet.

Homework : Section 14, P. 96 #1-3,11(a),13  
for 1(g), you may assume that words are anagrams of themselves.