Section 14 - Equivalence Classes

First do worksheet.

So, an equivalence relation partitions (divides up) a set into nonempty, pairwise disjoint (i.e., non-overlapping) sets that cover (i.e., union to) all of the set.

Def: If A is a set and R is an equivalence relation on A, then for each $a \in A$, the set $[a] = \{x \in A :_x R_a\}$ is called the equivalence class of a.

Ex: Let R be an equivalence relation on a set A. Let $a, b \in A$. Prove: ${}_aR_b$ if and only if $[a] = [b]$.

Proof: ($\Longrightarrow$) Hyp: ${}_aR_b$.
Let $x \in [a]$. Then ${}_xR_a$.
Since R is transitive, we have ${}_xR_b$.
Therefore $x \in [b]$.
So $[a] \subseteq [b]$.
Let $y \in [b]$. Then ${}_yR_b$.
Since R is symmetric, we have ${}_bR_y$.
Thus, since R is transitive, we have ${}_aR_y$.
Now, again because R is symmetric, we finally have ${}_yR_a$.
Hence $y \in [a]$.
So $[b] \subseteq [a]$.
Therefore, $[a] = [b]$.
($\Leftarrow$) Hyp: $[a] = [b]$.
Let $z \in [a]$. Then $z \in [b]$ also, since $[a] = [b]$.
Thus ${}_zR_a$ and ${}_zR_b$.
Since R is symmetric, we have ${}_aR_z$.
Therefore ${}_aR_b$, since R is transitive.

HW: Sec. 14, P. 96 #5, 6, 8, 9, 14