Sections 1 and 2 - Introduction, Definitions, and Divisibility

What is the purpose of this course?

It is an introduction to upper level math and CS. What distinguishes upper level from lower level courses? In earlier classes, you were probably concerned with getting the right answer. In this class we will be concerned much more with using correct logic.

For example, in Calc 1 you may have been asked to find the slope of the tangent line to a curve at a certain point on the curve. You would write a bunch of stuff, and most likely, as long as you had the correct number somewhere in your work, your professor would give you full or mostly full credit. Here, the stuff can't be "out of order" logically. You must proceed from step to step with full explanations in complete sentences. The point of this class is not just to learn some new mathematics (although we will). It is just as much to learn how to communicate the mathematics.

Be prepared to think harder and write longer than in any previous math class. Don't expect every homework assignment to fit on one page and every problem to fit on one line.

What does "discrete" mean?

First consider the real numbers (these are every number you can think of that is not imaginary, e.g., $5, \sqrt{7}, -10.4, \pi, \frac{4}{7}, \frac{e^{12}}{62}, 0$). Between any two real numbers you choose, there is another real number (in fact, infinitely many real numbers) between them. We call such a set continuous. Now consider the integers $(\ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots)$. Between consecutive integers (such as 2 and 3), there is no other integer. Thus, the integers are seperated from each other. We call this a discrete set. Note: Both continuous and discrete sets can be either finite or infinite. This class is based around the integers (unlike calculus which is based on the real numbers). So it is called Discrete Mathematics.

We are going to start with some common agreement about what we can assume. You should read App. D, P. 552-553 thoroughly, so you know what you can and cannot do without further justification. For example, if you know that a<b and b<c, then you may next conclude that a<c, without further justification.

From these basic facts, we can begin introducing some more advanced concepts.

Def (divisibility): Let a and b be integers. Then a is *divisible* by b (also b *divides* a, or b is a *factor* of a, or b is a *divisor* of a, or a is a *multiple* of b) if there exists an integer c such that $bc = a$.

If a is divisible by b, we may write $b|a$. Note: Do not confuse the sentence "$b|a$" with the quantity $\frac{b}{a}$ which is totally different. If anything, the quantity $\frac{a}{b}$ is more related to this concept than $\frac{b}{a}$, since as long as $b \neq 0$, we have $b|a$ whenever $\frac{a}{b}$ is an integer (c in the definition above).

Def (even): If a is an integer, then it is even if $2|a$. (Note: you may, and will often want to, replace "$2|a$" with the equivalent "there exists and integer x with $2x = a$".)

Def (odd): If a is an integer, then it is *odd* if there exists and integer x with $2x+1=a$.

Now, here's where we move into the frustrating territory for many of you. Most of you know that every integer is either even or odd, but not both. At this point you would probably like to assume that. But, we have not proved it yet. In this class, unless specifically instructed to do so, as with the stuff in App. D, you will not use anything that has not been formally proven.

Since we have not yet learned to prove anything formally, we'll have to wait to prove this statement.

Def (prime): An integer p is called *prime* if $p > 1$ and the only positive divisors of p are 1 and p.

Note: 2 is prime (and even). Some more primes are $5, 7, 11, 13, 17$, and so on. 1 is not prime.

Def (composite): An positive integer a is called *composite* if there exists an integer b such that $1 < b < a$ (that is $1 < b$ and $b < a$) and $b|a$.

Homework: Read Appendix D (P. 552-553).
Section 2 #1, 3,4 (hint: consider the quantity $a-b$),5,6,8 (a-e,h,i,l),9(a, b optional)