Section 23 - Proving a function is onto or one-to-one

Recall: If $f : A \to B$, then f is one-to-one if it satisfies the following property:

$$\text{If } c, d \in A \text{ and } f(c) = f(d) \text{ then } c = d.$$

Thus, if we want to prove that a function is one-to-one, the template for the proof is:

Proof: Hyp: Let $c, d \in A$ with $f(c) = f(d)$.
⋮
Conc: $c = d$.

Ex: Prove: $f : \mathbb{Z} \to \mathbb{Z}$ given by $f(x) = 3 - x$ is one-to-one.

Proof: Hyp: Let $c, d \in A$ with $f(c) = f(d)$.
Then $3 - c = 3 - d$
So $-c = -d$.
Conc: $c = d$.

Now, onto proofs are a bit trickier. Recall: If $f : A \to B$, then f is onto if it satisfies the following property:

$$\text{If } y \in B \text{ then } \exists x \in A \text{ such that } f(x) = y.$$

Thus, if we want to prove that a function is onto, the template for the proof is:

Proof: Hyp: Let $y \in B$.
Let $x = \ldots$
Then $x \in A$ because ...
And $f(x) = \ldots = y$.
Conc: $\exists x \in A$ such that $f(x) = y$.

The tricky bit is finding the x. This is not done within the proof. It is done with scratch work off to the side. Then it appears in the proof as if by magic.

Now do worksheet on this step.

Ex: Prove: $f : \mathbb{Z} \to \mathbb{Z}$ given by $f(x) = x + 4$ is onto.

Proof: Hyp: Let $y \in \mathbb{Z}$.
Let $x = y - 4$.
Then $x \in \mathbb{Z}$ by App. D.
And $f(x) = f(y - 4) = y - 4 + 4 = y$.
Conc: $\exists x \in \mathbb{Z}$ such that $f(x) = y$.

If you want to prove that a function is a bijection, you simply prove that it is one-to-one and onto.

Homework: Section23, P. 203 #9,13