Section 3 - Statements, Theorems, and Truth Tables

We call sentences statements.
Exs:

5 and 7 are both odd.
1 and 2 are both odd.
$8 < 9$
If it rains today I will bring my umbrella.
If $x < 4$ then $x < 7$.
I am going to the library today.
$\mathbb{Q} \subset \mathbb{R}$
Dogs are great.
What goes up must come down.

Note that statements can be either true, false, or undeterminable.

In Mathematics, truth is much more strict than in every day life. While we would most likely accept the final statement (or maybe even the last two) as true if it was said by somebody in ordinary conversation, in mathematics or CS, we would only accept it if it applied in every instance. That is, to accept the last two statements as true you would have to mean that every dog is great, without exception, and that everything that goes up will come down eventually. Just being almost always true is not enough.

Theorems

A theorem is a true statment that we have proved to be true.

A conjecture is a statement that has not been proved to be true or false (but is usually believed to be true by the person making the statement.)

We often state our theorems in what is called if-then form, because they are easier to prove that way. So, if we are asked to prove the statement "the sum of two even integers is even", we will begin by rewriting the statement as "If x and y are even then $x + y$ is even." and if asked to prove that "the reciprocal of a positive rational number is rational", we rewrite as "If q is a rational number then $\frac{1}{q}$ is rational."

Truth Tables

To begin our study of logic (which we will need to do proofs), we begin by studying what makes verious statements true or false. A basic statement such as "4 is even" or "it is raining" can be tested using the definitions. But we will be dealing with more complex statements that are built out of other statements using terms such as not, and, or , if, then, and so on.

So, if we have a statement we have named A, then not A will be a statement derived from A. For example, if A is the statement "4 is even", not A would be "4 is not even". And if A is the statement "it is raining", not A would be "it is not raining".

Here is the truth table for the statement "not A"

A	not A
T	F
F	T

Here is the truth table for the statement "A and B".

A	B	A and B
T	T	T
T	F	F
F	T	F
F	F	F

"A or B"

A	B	A or B
T	T	T
T	F	F
F	T	F
F	F	F

"If A then B"
(Use example "If it rains tomorrow, I will give you a ride home.")

A	B	If A then B
T	T	T
T	F	F
F	T	T (vacuous)
F	F	T (vacuous)

Thus, theorems of the form "If A then B" are true if the statement A being true forces the statement B to be true (we use the more gentle term implies.) So, to prove our if-then theorems, we will start by assuming A to be true and showing that B has to be true as well.

"A if and only if B"
First, we note that this is the same as "If A then B and if B then A"

A	B	A if and only if B
T	T	T
T	F	F
F	T	F
F	F	T

To prove our "A if and only if B" theorems, we will break it up into 2 if-then theorems and prove each seperately.

Vocabulary and Notation

"If A then B" can also be written as:

A implies B

B, if A

A, only if B

If not B then not A

A is sufficient for B

B is necessary for A

$A \implies B$

and it has some related statements:

original	If A then B
converse	If B then A (not logically the same as the original)
contrapositive	If not B then not A (logically the same as the original)
inverse	If B then A (not logically the same as the original)

"A if and only if B" can also be written as:

A iff B

A is necessary and sufficient for B

A is equivalent to B

$A \iff B$

Some other names for theorems are:

result

fact

proposition

lemma

corollary

claim

Homework: Section 3 #1-6