Section 7 - Lists

Recall that a set is an unordered collection of objects.

Def: A *list* is an ordered collection (sequence) of objects.

So, the sets $\{a, b, c\}$ and $\{b, a, c\}$ and $\{c, b, c, a\}$ are all identical. But the lists (a, b, c) and (b, a, c) and (c, b, c, a) are all different.

The *length* of the list is the number of elements.
A list of length 2 is called an *ordered pair*.
The list of length 0 is called the empty list, and denoted by $()$. This is not to be confused with the empty set which is written $\emptyset$.

Using the 26-letter alphabet, how many 3-letter lists can we make?

Method 1:

$$\begin{array}{llll} \text{AAA} & \text{AAB} & \dots & \text{AAZ} \\ \text{ABA} & \text{ABB} & \dots & \text{ABZ} \\ \vdots & & & \vdots \\ \text{AZA} & \text{AZB} & \dots & \text{AZZ} \\ \\ \text{BAA} & \text{BAB} & \dots & \text{BAZ} \\ \text{BBA} & \text{BBB} & \dots & \text{BBZ} \\ \vdots & & & \vdots \\ \text{BZA} & \text{BZB} & \dots & \text{BZZ} \\ \vdots \\ \text{ZAA} & \text{ZAB} & \dots & \text{ZAZ} \\ \text{ZBA} & \text{ZBB} & \dots & \text{ZBZ} \\ \vdots & & & \vdots \\ \text{ZZA} & \text{ZZB} & \dots & \text{ZZZ} \end{array}$$

Thus, we have $26 * (26 * 26) = 26^3 = 17,576$

Method 2 (The Multiplication Principle):

\# of ways to choose 1st letter $*$ # of ways to choose 2nd letter $*$ # of ways to choose 3rd letter $=$ $26 * 26 * 26 = 26^3 = 17,576$

Now suppose that we don't wish to allow repeats.

$$26 * 25 * 24 = 15,600$$

What if we want a code that is made up of 2 letters followed by 4 digits?

$$26 * 26 * 10 * 10 * 10 * 10$$

If no repeats are allowed?

$$26 * 25 * 10 * 9 * 8 * 7$$

How about 3 letters, with each letter being allowed to be used no more than twice.
Method 1:

$$\begin{aligned}
& \text{\# of possible lists with no repeats} + \text{\# of possible lists with the first letter repeated in pos 2}+ \\
& \text{\# of possible lists with the first letter repeated in pos 3}+\text{\# of possible lists with the second letter repeated} \\
= \ & 26 * 25 * 24 + 26 * 1 * 25 + 26 * 25 * 1 + 26 * 25 * 1 = 17,550
\end{aligned}$$

Method 2:

$$\begin{aligned}
& \text{\# of possible lists - \# of possible lists with all letters the same} \\
= \ & 26 * 26 * 26 - 26 * 1 * 1 = 17,576 - 26 = 17,550
\end{aligned}$$

Homework: Section 7, P. 43 $\#1, 2, 4, 5, 6, 9, 11, 14, 15$