Section 8 - Factorial Notation

Recall series notation for representing sums:

$$\begin{aligned}\sum_{i=2}^{5}\left(i^2+1\right) &= \left(2^2+1\right)+\left(3^2+1\right)+\left(4^2+1\right)+\left(5^2+1\right)\\ &= 5+10+17+26\\ &= 58\end{aligned}$$

If we want to do the same thing with products, we use $\prod$ instead of $\sum$ :

$$\begin{aligned}\prod_{i=2}^{5}\left(i^2+1\right) &= \left(2^2+1\right)*\left(3^2+1\right)*\left(4^2+1\right)*\left(5^2+1\right)\\ &= 5*10*17*26\\ &= 22100\end{aligned}$$

Now, very often in math, we with to form the product of many consecutive numbers. For example, if we want to know how many ways we can arrange the letters of the alphabet, we get

$$26\cdot 25\cdot 24\cdot \ldots \cdot 3\cdot 2\cdot 1=\prod_{k=1}^{26} k = k!$$

where the last is pronounced "k factorial".

For example, in #7.13, you are asked in how many ways 20 books can be arranged on a shelf. The answer can be written as 20!.

Now, let's consider another list problem.

How many 10-letter lists can be made if no repeats are allowed?

Ans: $26*25*24*23*22*21*20*19*18*17$

We could write this using product notation as:

$$\prod_{k=17}^{26} k$$

but let's figure out how to write this using factorial notation.

$$\begin{aligned}26*25*24*23*22*21*20*19*18*17 &= \frac{(26*25*24*23*22*21*20*19*18*17)*(16*15*\ldots*3*}{(16*15*\ldots*3*2*1)}\\ &= \frac{26!}{16!}\ \text{(note:}\ = \frac{26!}{(26-10)!}\end{aligned}$$

Your book also uses the following notation as well:

$$\begin{aligned}(n)_k &= \frac{n!}{(n-k)!}\\ (n)_n &= n!\end{aligned}$$

For convenience, we define 0! to be 1. (which makes the following formulas above match up)

Homework: Section 8, P. 48 #1, 4, 5, 7