Section 9a - Introduction to Sets

Def: A *set* is an unordered collection of objects (elements).

We use {} to denote sets when writing out the elements..

Exs:

$$\{1, 2, 3, 4, 5\}$$
$$\{AUD\}$$
$$\mathbb{Z}, \text{ etc.}$$

There are some very big differences between lists and sets. For sets, reordering or repeating an element does not change the set, unlike for lists. So, for example$\{a, b, c\}$ and $\{b, a, c\}$ and $\{c, b, c, a\}$ are all the same set.

We write $\in$ to mean “is an element of”.

$$\begin{aligned} A &\in \{A, B, C\}. \\ 5 &\in \mathbb{Z}. \end{aligned}$$

We also write $\in$ to mean "be an element of" (or “be in” for short”)

$$\text{Let } x \in \mathbb{Z}.$$

similarly, we use $\notin$ to mean “is not an element of” or “not be an element of”.

The empty set is the set with no elements. We denote it by $\emptyset$ or $\{\}$.
The cardinatlity of a set is the number of distinct elements. We use || to denote cardinality.

$$\begin{aligned} |\{1, 4, 7\}| &= 3 \\ |\{1, 4, 4, 7, 1\}| &= 3 \\ |\mathbb{Z}| &= \infty \\ |\emptyset| &= 0 \end{aligned}$$

Suppose that we want to discuss the set of positive even integers. There are several additional ways to descrive this set:

$\{2, 4, 6, 8, \ldots\}$

$\mathbb{Z}^{\text{even},+}$

“The set of elements x such that x is an integer and x is even”

$\{x : x \in \mathbb{Z} \text{ and } x \text{ is even}\}$

$\{x : x \in \mathbb{Z} \text{ and } 2 \mid x\}$

“The set of integers x such that x is even.”

$\{x \in \mathbb{Z} : 2 \mid x\}$

Note, there is nothing special about x. It is called a “dummy variable” or “place holder variable”.

$$\{x \in \mathbb{Z} : 2 \mid x\} = \{w \in \mathbb{Z} : 2 \mid w\} \text{ etc.}$$

Ex: Write out the following set by listing its elements between curly braces and determine its cardinatlity.

$$\{x \in \mathbb{Z}: x > -5 \text{ and } x \leq 3\}$$

Ans:

$$\{-4, -3, -2, -1, 0, 1, 2, 3\}$$

$$\text{Thus } |\{x \in \mathbb{Z}: x > -5 \text{ and } x \leq 3\}| = 8$$

Sets of sets. We can make sets out of anything. Therefore, we can have sets of sets.

$\{\{2,3,5,9,10\},\{2,4,7,8\},\{1\}\}$ is a perfectly fine set. It has cardinatlity 3 and
$\{2,4,7,8\} \in \{\{2,3,5,9,10\},\{2,4,7,8\},\{1\}\}$,
$\{1\} \in \{\{2,3,5,9,10\},\{2,4,7,8\},\{1\}\}$,
but $1 \notin \{\{2,3,5,9,10\},\{2,4,7,8\},\{1\}\}$.

Def: Let A and B be sets. Then A is a *subset* of B if every element of A is also an element of B. We write $A \subseteq B$.

Ex.

$$\begin{aligned}
\{3,5\} &\subseteq \{1,2,3,4,5\} \\
\{1,2,3,4\} &\subseteq \{1,2,3,4,5\} \\
\{1,2,3,4,5\} &\subseteq \{1,2,3,4,5\} \\
\emptyset &\subseteq \{1,2,3,4,5\}
\end{aligned}$$

Note: If $A \subseteq B$ and $x \in A$ then $x \in B$.

Let's find all of the subsets of $\{a,b,c,d\}$.

$$\begin{aligned}
\text{Ans} \quad : \quad & \emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{b,d\}, \{c,d\}, \\
& \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}
\end{aligned}$$

If we make the set that contains all of these subsets, we get $\{\emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{$
a set with 16 elements. This set is called the *power set* of $\{a,b,c,d\}$ and is denoted by $2^{\{a,b,c,d\}}$. So if you see 2 raised to a set, you should immediately put a line through it and rewrite it as "the power set of [the set]" or "the set whose elements are the subsets of [the set]".

Homework: Section 9, P. 57 #1,2,3