Math 301 Quiz #2a Name: *KEY*

1. Let x be an integer. Prove: x is even if and only if $x-3$ is odd.

Proof: Let x be an integer.

($\Longrightarrow$) Hypothesis: x is even.
$x = 2a$ for some $a \in \mathbb{Z}$.
$x - 3 = 2a - 3 = 2\,(a-2)+1$
and $a - 2 \in \mathbb{Z}$ by the closure property of integer addition.
Conclusion: $x - 3$ is odd.

($\Longleftarrow$) Hypothesis: $x - 3$ is odd.
$x - 3 = 2b + 1$ for some $b \in \mathbb{Z}$.
$x = 2b + 4 = 2\,(b+2)$
and $b + 2 \in \mathbb{Z}$ by the closure property of integer addition.
Conclusion: x is even.

2. Disprove: If x is odd, then x is prime.

A counterexample is provided by letting $x = 9$.
9 is odd since $9 = 2(4)+1$, but not prime since $3 \mid 9$.

3. Prove that $x \to (y \vee z)$ is logically equivalent to $(x \wedge \neg\, z) \to y$.

x	y	z	$y \vee z$	$x \to (y \vee z)$	$\neg z$	$x \wedge \neg z$	$(x \wedge \neg\, z) \to y$
T	T	T	T	T	F	F	T
T	T	F	T	T	T	T	T
T	F	T	T	T	F	F	T
T	F	F	F	F	T	T	F
F	T	T	T	T	F	F	T
F	T	F	T	T	T	F	T
F	F	T	T	T	F	F	T
F	F	F	F	T	T	F	T

Math 301 Quiz #2b Name: *KEY*

1. Let x be an integer. Prove: x is even if and only if $x-5$ is odd.

Proof: Let x be an integer.

($\Longrightarrow$) Hypothesis: x is even.
$x = 2a$ for some $a \in \mathbb{Z}$.
$x - 5 = 2a - 5 = 2(a-3)+1$
and $a - 3 \in \mathbb{Z}$ by the closure property of integer addition.
Conclusion: $x - 5$ is odd.

($\Longleftarrow$) Hypothesis: $x - 5$ is odd.
$x - 5 = 2b + 1$ for some $b \in \mathbb{Z}$.
$x = 2b + 6 = 2(b+3)$
and $b + 3 \in \mathbb{Z}$ by the closure property of integer addition.
Conclusion: x is even.

2. Disprove: If x is odd, then x is prime.

$$\begin{aligned} \text{A counterexample is provided by letting } x &= 9. \\ 9 \text{ is odd since } 9 &= 2(4)+1, \text{ but not prime since } 3 \mid 9. \end{aligned}$$

3. Prove that $x \to (y \vee z)$ is logically equivalent to $(x \wedge \neg y) \to z$.

x	y	z	$y \vee z$	$x \to (y \vee z)$	$\neg y$	$x \wedge \neg y$	$(x \wedge \neg y) \to z$
T	T	T	T	T	F	F	T
T	T	F	T	T	F	F	T
T	F	T	T	T	T	T	T
T	F	F	F	F	T	T	F
F	T	T	T	T	F	F	T
F	T	F	T	T	F	F	T
F	F	T	T	T	T	F	T
F	F	F	F	T	T	F	T