Math 301 Quiz #5a Name: KEY

1. Let A, B, C be sets. Prove: If $A \subseteq B$, then $C - B \subseteq C - A$.

$$\begin{aligned} \text{Hyp} &: \quad A \subseteq B \\ & \quad \text{Let } x \in C - B. \\ & \quad \text{Then } x \in C \text{ and } x \notin B. \\ & \quad \text{If } x \in A, \text{ then } x \in B \text{ since } A \subseteq B. \\ & \quad \text{Thus, since } x \notin B, x \notin A. \\ & \quad \text{Therefore } x \in C - A. \\ \text{Conc} &: \quad C - B \subseteq C - A \end{aligned}$$

2. Let A, B, C be sets. Prove: $A \times B \subseteq A \times (B \cup C)$.

$$\begin{aligned} & \text{Let } x \in A \times B. \\ & \text{Then } x = (c, d) \text{ with } c \in A \text{ and } d \in B. \\ & \text{Since } d \in B,\ d \in B \cup C. \\ & \text{So } x \in A \times (B \cup C). \\ \text{Conc} :\ & A \times B \subseteq A \times (B \cup C) \end{aligned}$$

3. Let R be the relation on integers given by $R = \{(x, y) : x, y \in \mathbb{Z}, (x+1) \mid y\}$. Determine if the following are TRUE or FALSE. (As always, **do not use T and F**.)

(a) ${}_2R_5$.

False, because $(2+1) \nmid 5$.

(b) ${}_6R_{14}$.

True, because $(6+1) \mid 14$.

(c) If ${}_aR_b$ then ${}_aR_b^{-1}$.

False.
Counterexample: ${}_6R_{14}$ is true, but ${}_6R_{14}^{-1}$ is false.

(d) ${}_{10}R_4^{-1}$.

True, because ${}_4R_{10}$.