Math 301 Quiz #6 Name: KEY

This quiz is due Monday, April 6, by noon. If you do not turn it in in class, you may drop it off in my office, DA 219.

If you have questions about the problems on the quiz, you may ask me them, but, depending on the questions, I may refuse to answer all or part. **Your work must be completely your own. You are not permitted to discuss any of the problems on this quiz with any other person (except me) inside or outside of our class.**

1. Let R be a relation on Z given by ${}_aR_b$ if $a^2 \mid b$.

   (a) Prove that R is transitive.

   Proof: Let $x, y, z \in \mathbb{Z}$ so that ${}_xR_y$ and ${}_yR_z$.
   Then $x^2 \mid y$ and $y^2 \mid z$.
   So $y = x^2a$ and $z = y^2b$ for some $a, b \in \mathbb{Z}$.
   Thus $z = (x^2a)^2 b = x^2 (x^2a^2b)$ and $x^2a^2b \in \mathbb{Z}$.
   So $x^2 \mid z$.
   Therefore ${}_xR_z$.
   Conc: R is transitive.

   (b) Provide a counterexample to show that R is not symmetric.

   $$2^2 \mid 8, \text{ but } 8^2 \nmid 2. \text{ So } {}_2R_8 \text{ but } {}_8\not R_2.$$

2. For each set A and relation R given, circle all of properties that R has.

   (a) $A = \{1, 2, 3\}, R = \{(1,1), (1,2), (2,1), (2,2), (3,3)\}$.

   [reflexive] irreflexive [symmetric] antisymmetric [transitive] [equivalence relation]

   (b) $A = \{1, 2, 3\}, R = \{(1,2), (2,1)\}$.

   reflexive [irreflexive] [symmetric] antisymmetric transitive equivalence relation

   (c) $A = \mathbb{Z}$, R is given by ${}_aR_b$ if $a < b$.

   reflexive [irreflexive] symmetric [antisymmetric] [transitive] equivalence relation

3. Find 3 different integers, k, so that $k \equiv 3 \pmod 7$

   $$k = 10, k = 17, k = 24$$
   (many other answers possible)