Math 341 Extra Credit for Exam 2 Name:: KEY

Show all work. Your answers must be fully justified.

For the following differential equation, let $y(x) = \sum_{n=0}^{\infty} a_n x^n$ be a series solution.

$$\left(2x^2+3\right) y'' + 5xy' - 2y = 0$$

1. Find a recurrence relation for the coefficients a_n.

$$\begin{aligned} y'(x) &= \sum_{n=1}^{\infty} n a_n x^{n-1} \\ y''(x) &= \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2} \end{aligned}$$

$$\begin{aligned} \left(2x^2+3\right) \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2} + 5x \sum_{n=1}^{\infty} n a_n x^{n-1} - 2 \sum_{n=0}^{\infty} a_n x^n &= 0 \\ \sum_{n=2}^{\infty} 2n(n-1) a_n x^n + \sum_{n=2}^{\infty} 3n(n-1) a_n x^{n-2} + \sum_{n=1}^{\infty} 5n a_n x^n - \sum_{n=0}^{\infty} 2a_n x^n &= 0 \\ \sum_{m=2}^{\infty} 2m(m-1) a_m x^m + \sum_{m=0}^{\infty} 3(m+2)(m+1) a_{m+2} x^m + \sum_{m=1}^{\infty} 5m a_m x^m - \sum_{m=0}^{\infty} 2a_m x^m &= 0 \end{aligned}$$

$$m = 0: \quad 3(0+2)(0+1) a_{0+2} - 2a_0 = 0 \quad \Longrightarrow \quad \boxed{a_2 = \frac{1}{3} a_0}$$

$$m = 1: \quad 3(1+2)(1+1) a_{1+2} + 5(1) a_1 - 2a_1 = 0 \quad \Longrightarrow \quad \boxed{a_3 = -\frac{1}{6} a_1}$$

$$m \geq 2: \quad 2m(m-1) a_m + 3(m+2)(m+1) a_{m+2} + 5m a_m - 2a_m = 0$$

$$a_{m+2} = \frac{2 - 5m - 2m(m-1)}{3(m+2)(m+1)} a_m$$

$$\boxed{a_{m+2} = \frac{2 - 3m - 2m^2}{3(m+2)(m+1)} a_m, \qquad m \geq 2}$$

2. If $a_0 = 1$ and $a_1 = 2$, find a_2, a_3 and a_4.

$$\begin{aligned} a_2 &= \frac{1}{3} a_0 = \frac{1}{3}(1) = \frac{1}{3} \\ a_3 &= -\frac{1}{6} a_1 = -\frac{1}{6}(2) = -\frac{1}{3} \\ a_4 &= \frac{2 - 3(2) - 2(2)^2}{3(2+2)(2+1)} a_2 = -\frac{1}{3}\left(\frac{1}{3}\right) = -\frac{1}{9} \end{aligned}$$