Math 341 Exam 2
April 30, 2009

Name___________________________

Show all of your work.
No calculators permitted.
Do not leave any answers blank.

1. Solve the initial value problem.

$$y'' + 3y' + 2y = 0, \qquad y(0) = 5, \qquad y'(0) = -3$$

2. (a) Demonstrate that the following set of functions is linearly independent by computing a Wronskian:

$$\left\{te^t, e^{-t}, 1\right\}$$

(b) Demonstrate that the following set of functions is linearly dependent without computing a Wronskian:

$$\left\{\sin^2 t, \cos^2 t, t^2, 3\right\}$$

3. Consider the differential equation

$$t^2 y'' + 2ty' - 2y = 0,\ t > 0$$

Use the fact that $y_1(t) = \frac{1}{t^2}$ is a solution to this equation and the method of Reduction of Order to find a second linearly independent solution of the form $y_2(t) = v(t) \cdot y_1(t)$.

4. Solve the following differential equation using the method of Undetermined Coefficients.

$$y'' - y = te^{3t}$$

5. Solve the following differential equation using the Method of Variation of Parameters.

$$y'' - 6y' + 9y = \frac{e^{3t}}{t^3}$$

6. (a) Find the general solution $y(t)$ for the sixth order homogeneous linear differential equation with constant coefficients that has the following characteristic equation:

$$(r-2)^4 \left(2r^2 - r + 1\right) = 0$$

(b) Use the Method of Undetermined Coefficients to find the form of $y(t)$ so that $y(t)$ is a solution to the following differential equation. (Do not solve for the coefficients!)

$$y''' + y'' = 1 + \sin 2t$$

7. For the following differential equation, let $y(x) = \sum_{n=0}^{\infty} a_n x^n$ be a series solution.

$$4xy'' + xy' + 3x^2 y = 0$$

(a) Find a recurrence relation for the coefficients a_n.

(b) If $a_0 = 1$ and $a_1 = 2$, find a_2 and a_3.