

Section 1.3

In each of Problems 1 through 6 determine the order of the given differential equation; also state whether the equation is linear or nonlinear.

1. $t^2 \frac{d^2 y}{dt^2} + t \frac{dy}{dt} + 2y = \sin t$
2. $(1 + y^2) \frac{d^2 y}{dt^2} + t \frac{dy}{dt} + y = e^t$
3. $\frac{d^4 y}{dt^4} + \frac{d^3 y}{dt^3} + \frac{d^2 y}{dt^2} + \frac{dy}{dt} + y = 1$
4. $\frac{dy}{dt} + ty^2 = 0$
5. $\frac{d^2 y}{dt^2} + \sin(t + y) = \sin t$
6. $\frac{d^3 y}{dt^3} + t \frac{dy}{dt} + (\cos^2 t)y = t^3$

In each of Problems 7 through 14 verify that each given function is a solution of the differential equation.

7. $y'' - y = 0$; $y_1(t) = e^t$, $y_2(t) = \cosh t$
8. $y'' + 2y' - 3y = 0$; $y_1(t) = e^{-3t}$, $y_2(t) = e^t$

9. $ty' - y = t^2$; $y = 3t + t^2$
10. $y''' + 4y'' + 3y = t$; $y_1(t) = t/3$, $y_2(t) = e^{-t} + t/3$
11. $2t^2 y'' + 3ty' - y = 0$, $t > 0$; $y_1(t) = t^{1/2}$, $y_2(t) = t^{-1}$
12. $t^2 y'' + 5ty' + 4y = 0$, $t > 0$; $y_1(t) = t^{-2}$, $y_2(t) = t^{-2} \ln t$
13. $y'' + y = \sec t$, $0 < t < \pi/2$; $y = (\cos t) \ln \cos t + t \sin t$
14. $y' - 2ty = 1$; $y = e^{t^2} \int_0^t e^{-s^2} ds + e^{t^2}$

In each of Problems 15 through 18 determine the values of r for which the given differential equation has solutions of the form $y = e^{rt}$.

15. $y' + 2y = 0$
16. $y'' - y = 0$
17. $y'' + y' - 6y = 0$
18. $y''' - 3y'' + 2y' = 0$

In each of Problems 19 and 20 determine the values of r for which the given differential equation has solutions of the form $y = t^r$ for $t > 0$.

19. $t^2 y'' + 4ty' + 2y = 0$
20. $t^2 y'' - 4ty' + 4y = 0$

In each of Problems 21 through 24 determine the order of the given partial differential equation; also state whether the equation is linear or nonlinear. Partial derivatives are denoted by subscripts.

21. $u_{xx} + u_{yy} + u_{zz} = 0$
22. $u_{xx} + u_{yy} + uu_x + uu_y + u = 0$
23. $u_{xxxx} + 2u_{xxyy} + u_{yyyy} = 0$
24. $u_t + uu_x = 1 + u_{xx}$