

1. Solve each of the following initial value problems and plot the solutions for several values of y_0 . Then describe in a few words how the solutions resemble, and differ from, each other.

(a) $dy/dt = -y + 5, \quad y(0) = y_0$

(b) $dy/dt = -2y + 5, \quad y(0) = y_0$

(c) $dy/dt = -2y + 10, \quad y(0) = y_0$

2. Follow the instructions for Problem 1 for the following initial value problems:

(a) $dy/dt = y - 5, \quad y(0) = y_0$

(b) $dy/dt = 2y - 5, \quad y(0) = y_0$

(c) $dy/dt = 2y - 10, \quad y(0) = y_0$

3. Consider the differential equation

$$dy/dt = -ay + b,$$

where both a and b are positive numbers.

(a) Solve the differential equation.

(b) Sketch the solution for several different initial conditions.

(c) Describe how the solutions change under each of the following conditions:

i. a increases.

ii. b increases.

iii. Both a and b increase, but the ratio b/a remains the same.

7. The field mouse population in Example 1 satisfies the differential equation

$$dp/dt = 0.5p - 450.$$

(a) Find the time at which the population becomes extinct if $p(0) = 850$.

(b) Find the time of extinction if $p(0) = p_0$, where $0 < p_0 < 900$.

(c) Find the initial population p_0 if the population is to become extinct in 1 year.

15. According to Newton's law of cooling (see Problem 23 of Section 1.1), the temperature $u(t)$ of an object satisfies the differential equation

$$du/dt = -k(u - T),$$

where T is the constant ambient temperature and k is a positive constant. Suppose that the initial temperature of the object is $u(0) = u_0$.

(a) Find the temperature of the object at any time.

(b) Let τ be the time at which the initial temperature difference $u_0 - T$ has been reduced by half. Find the relation between k and τ .

17. Consider an electric circuit containing a capacitor, resistor, and battery; see Figure 1.2.3. The charge $Q(t)$ on the capacitor satisfies the equation³

$$R \frac{dQ}{dt} + \frac{Q}{C} = V,$$

where R is the resistance, C is the capacitance, and V is the constant voltage supplied by the battery.

- If $Q(0) = 0$, find $Q(t)$ at any time t , and sketch the graph of Q versus t .
- Find the limiting value Q_L that $Q(t)$ approaches after a long time.
- Suppose that $Q(t_1) = Q_L$ and that at time $t = t_1$ the battery is removed and the circuit closed again. Find $Q(t)$ for $t > t_1$ and sketch its graph.

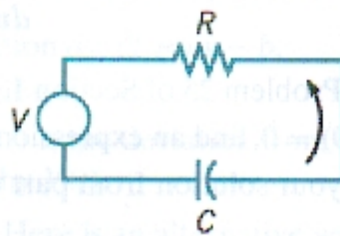


FIGURE 1.2.3 The electric circuit of Problem 17.

18. A pond containing 1,000,000 gal of water is initially free of a certain undesirable chemical (see Problem 21 of Section 1.1). Water containing 0.01 g/gal of the chemical flows into the pond at a rate of 300 gal/hr, and water also flows out of the pond at the same rate. Assume that the chemical is uniformly distributed throughout the pond.
- Let $Q(t)$ be the amount of the chemical in the pond at time t . Write down an initial value problem for $Q(t)$.
 - Solve the problem in part (a) for $Q(t)$. How much chemical is in the pond after 1 year?
 - At the end of 1 year the source of the chemical in the pond is removed; thereafter pure water flows into the pond, and the mixture flows out at the same rate as before. Write down the initial value problem that describes this new situation.
 - Solve the initial value problem in part (c). How much chemical remains in the pond after 1 additional year (2 years from the beginning of the problem)?
 - How long does it take for $Q(t)$ to be reduced to 10 g?
 - Plot $Q(t)$ versus t for 3 years.