

Show all work. Your answers must be fully justified.

1. For each differential equation, circle linear or non-linear as appropriate and determine the order.

(a)  $9 \frac{d^3 y}{dt^3} y^2 - 3yt = 4t^7 + 2t$       linear / non-linear      order: 3

(b)  $2 \frac{d^2 y}{dt^2} t^5 - 4yt = 5y + 2$       linear / non-linear      order: 2

2. Verify that  $y = (\sin t)^2$  is a solution to

$$y'' + 4y = 2.$$

(It may be helpful to recall the Pythagorean Identity:  $(\cos t)^2 + (\sin t)^2 = 1$ )

$$\begin{aligned} y' &= 2 \sin t \cos t \\ y'' &= 2 (\cos t)^2 - 2 (\sin t)^2 \end{aligned}$$

$$\begin{aligned} \text{LHS} = y'' + 4y &= 2 (\cos t)^2 - 2 (\sin t)^2 + 4 (\sin t)^2 \\ &= 2 (\cos t)^2 + 2 (\sin t)^2 \\ &= 2 ((\cos t)^2 + (\sin t)^2) \\ &= 2 = \text{RHS} \end{aligned}$$

3. Find the general (explicit) solution to

$$y' - 2y = 3e^t$$

$$u(t) = e^{\int -2 dt} = e^{-2t}$$

$$y' e^{-2t} - 2e^{-2t} y = 3e^{-t}$$

$$\frac{d}{dt} (e^{-2t} y) = 3e^{-t}$$

$$e^{-2t} y = -3e^{-t} + C$$

$$y = -3e^t + Ce^{2t}$$