Math 341 Quiz #3 Name: KEY

Show all work. Your answers must be fully justified.

1. Solve the initial value problem. (You may use either the Integrating Factor method or the Seperation of Variables Method.)

$$\frac{dy}{dt} = 4y - 6, \qquad y(0) = 2.$$

Integrating Factor Method: $\frac{dy}{dt} - 4y = -6$

$$u(t) = e^{\int -4\, dt} = e^{-4t}$$

$$\begin{aligned} \frac{dy}{dt}e^{-4t} - 4ye^{-4t} &= -6e^{-4t} \\ \frac{d}{dt}\left(e^{-4t}y\right) &= -6e^{-4t} \\ e^{-4t}y &= \frac{3}{2}e^{-4t} + C \\ y &= \frac{3}{2} + Ce^{4t} \end{aligned}$$

$$\begin{aligned} \text{Putting in } y(0) &= 2 \\ 2 &= \frac{3}{2} + C \\ C &= \frac{1}{2} \end{aligned}$$

Seperation of Variables Method: $\frac{dy}{4y-6} = dt$

$$\begin{aligned} \int \frac{dy}{4y-6} &= \int dt \\ \frac{1}{4}\ln(4y-6) &= t + C \\ \ln(4y-6) &= 4t + 4C \\ 4y - 6 &= e^{4t+4C} \\ 4y &= e^{4C}e^{4t} + 6 \\ y &= \frac{1}{4}e^{4C}e^{4t} + \frac{3}{2} \end{aligned}$$

$$\begin{aligned} \text{Putting in } y(0) &= 2 \\ 2 &= \frac{1}{4}e^{4C} + \frac{3}{2} \\ \frac{1}{4}e^{4C} &= \frac{1}{2} \end{aligned}$$

So, with either method, we get the answer :

$$y = \frac{3}{2} + \frac{1}{2}e^{4t}$$

2. Solve the differential equation (explicitly)

$$\frac{dy}{dx} = 6x^2y^4.$$

$$\begin{aligned}
\frac{dy}{y^4} &= 6x^2dx \\
\int y^{-4}dy &= \int 6x^2dx \\
-\frac{1}{3}y^{-3} &= 2x^3 + C \\
\frac{1}{y^3} &= -6x^3 - 3C \\
y^3 &= \frac{1}{-6x^3 - 3C} \\
y &= \sqrt[3]{\frac{1}{-6x^3 - 3C}}
\end{aligned}$$

(Note: Several other representations are fine, such as $y = \sqrt[3]{\frac{1}{-6x^3 + C}}$ or $y = -\frac{1}{\sqrt[3]{6x^3 + 3C}}$, etc.)