

Show all work. Your answers must be fully justified.

1. A tank holds 200 liters of pure water. The water is accidentally contaminated with 15 grams of a chemical. You may assume the chemical adds a negligible amount of volume (i.e., the volume of liquid in the tank is still 200 liters). To remove the chemical, fresh water is pumped in at a rate of 5 liters/min and the contaminated water is pumped out at the same rate. Set up an initial value problem to model this situation..

$y(t)$ = the amount of chemical in the pool after t minutes of pumping.

$$\begin{aligned}\frac{dy}{dt} &= \text{rate chemical is coming in} - \text{rate it is going out} \\ \frac{dy}{dt} &= (0)(5) - \left(\frac{y}{200}\right)(5)\end{aligned}$$

So, the initial value problem is :

$$\frac{dy}{dt} = -\frac{y}{40}, \quad y(0) = 15$$

2. A person invests \$5000 in a savings account that has an annual interest rate of 3.5% compounded continuously. Each year at the same time, the person withdraws \$50 from the account. Set up an initial value problem to model this situation.

$S(t)$ = the amount in the account after t years.

$$\begin{aligned}\frac{dS}{dt} &= \text{rate money is added} - \text{rate money is withdrawn} \\ \frac{dS}{dt} &= (0.035)S - 50\end{aligned}$$

So, the initial value problem is :

$$\frac{dS}{dt} = 0.035 S - 50, \quad y(0) = 5000$$