Math 341 Quiz #6 Name: KEY

Show all work. Your answers must be fully justified.

1. Find the general solution to the differential equation.

(a) $y'' + y' + 3y = 0$

$$\begin{aligned} r^2 + r + 3 &= 0 \\ r &= \frac{-1 \pm \sqrt{1-12}}{2} = -\frac{1}{2} \pm i\frac{\sqrt{11}}{2} \end{aligned}$$

$$y(t) = c_1 e^{-t/2} \cos \frac{t\sqrt{11}}{2} + c_2 e^{-t/2} \sin \frac{t\sqrt{11}}{2}$$

(b) $y'' - 2y' - 15y = 0$

$$\begin{aligned} r^2 - 2r - 15 &= 0 \\ (r-5)(r+3) &= 0 \\ r &= 5, -3 \end{aligned}$$

$$y(t) = c_1 e^{5t} + c_2 e^{-3t}$$

2. Consider the differential equation

$$t^2 y'' - t(t+2) y' + (t+2) y = 0$$

(a) Use the fact that $y_1(t) = t$ is a solution to this equation to find a second linearly independent solution of the form $y_2(t) = v(t) \cdot y_1(t)$.

Let $y(t) = tv(t)$

$$\begin{aligned} \text{Then } t^2 y'' - t(t+2) y' + (t+2) y &= t^2 (2v'(t) + tv''(t)) - t(t+2)(v(t) + tv'(t)) + (t+2) tv(t) \\ &= t^3 v''(t) - t^3 v'(t) \end{aligned}$$

Setting equal to 0, we get :

$$\begin{aligned} t^3 v''(t) &= t^3 v'(t) \\ \frac{v''(t)}{v'(t)} &= 1 \\ \ln v'(t) &= t \\ v'(t) &= e^t \\ v(t) &= e^t \end{aligned}$$

$$y_2(t) = te^t$$