Math 341 Quiz #7 Name:________________________

Show all work. Your answers must be fully justified.

1. A seventh order homogeneous linear differential equation with constant coefficients has a characteristic equation that factors as follows:

$$\left(r^2 + 2r + 5\right)(2r - 1)^3 (r - 1)(r + 9) = 0$$

Find the general solution $y(t)$.

2. Show that the following set of functions is linearly independent:

$$\left\{2t, \cos t, e^t\right\}$$

3. Show that the following set of functions is linearly dependent without calculating a Wronskian.

$$\left\{3t^2 + 5t, t, t^2\right\}$$

TABLE 3.6.1 The Particular Solution of $ay'' + by' + cy = g_i(t)$

$g_i(t)$	$Y_i(t)$
$P_n(t) = a_0t^n + a_1t^{n-1} + \cdots + a_n$	$t^s(A_0t^n + A_1t^{n-1} + \cdots + A_n)$
$P_n(t)e^{\alpha t}$	$t^s(A_0t^n + A_1t^{n-1} + \cdots + A_n)e^{\alpha t}$
$P_n(t)e^{\alpha t}\begin{cases}\sin\beta t\\ \cos\beta t\end{cases}$	$t^s[(A_0t^n + A_1t^{n-1} + \cdots + A_n)e^{\alpha t}\cos\beta t$ $+ (B_0t^n + B_1t^{n-1} + \cdots + B_n)e^{\alpha t}\sin\beta t]$

Notes. Here s is the smallest nonnegative integer ($s = 0, 1$, or 2) that will ensure that no term in $Y_i(t)$ is a solution of the corresponding homogeneous equation. Equivalently, for the three cases, s is the number of times 0 is a root of the characteristic equation, α is a root of the characteristic equation, and $\alpha + i\beta$ is a root of the characteristic equation, respectively.

4. Consider the following differential equation:

$$y'' - y' - 2y = g(t)$$

(a) Find $y_c(t)$. (That is, find $y(t)$ when $g(t) = 0$.)

(b) Find $y(t)$ if $g(t) = 3e^{-t}$.

(c) Find the form of $y(t)$ if $g(t) = t^3 - 9te^{5t} \sin 2t$. (Do not solve for the coefficients!)