Math 341 Quiz #7 Name: KEY

Show all work. Your answers must be fully justified.

1. A seventh order homogeneous linear differential equation with constant coefficients has a characteristic equation that factors as follows:

$$\left(r^2+2r+5\right)(2r-1)^3(r-1)(r+9)=0$$

Find the general solution $y(t)$.

$$r=1,-9,\frac{1}{2},\frac{1}{2},\frac{1}{2},-1\pm 2i$$

$$y(t)=c_1e^t+c_2e^{-9t}+c_3e^{t/2}+c_4te^{t/2}+c_5t^2e^{t/2}+c_6e^{-t}\cos 2t+c_7e^{-t}\sin 2t$$

2. Show that the following set of functions is linearly independent:

$$\left\{2t,\cos t,e^t\right\}$$

$$W=\begin{vmatrix} 2t & \cos t & e^t \\ 0 & -\sin t & e^t \\ 0 & -\cos t & e^t \end{vmatrix}=2te^t(\cos t-\sin t)$$

So the Wronskian is not identically 0.

3. Show that the following set of functions is linearly dependent without calculating a Wronskian.

$$\left\{3t^2+5t,t,t^2\right\}$$

$$(1)\left(3t^2+5t\right)+(-5)t+(-3)t^2=0$$

4. Consider the following differential equation:

$$y'' - y' - 2y = g(t)$$

(a) Find $y_c(t)$.

$$\begin{aligned} r^2 - r - 2 &= 0 \\ (r-2)(r+1) &= 0 \\ r &= 2, -1 \end{aligned}$$

$$y_c(t) = c_1 e^{2t} + c_2 e^{-t}$$

(b) Find $y(t)$ if $g(t) = 3e^{-t}$.

$$Y(t) = Ate^{-t}$$

$$\begin{aligned} Y' &= Ae^{-t} - Ate^{-t} \\ Y'' &= -2Ae^{-t} + Ate^{-t} \end{aligned}$$

$$\begin{aligned} Ate^{-t} - 2Ae^{-t} - Ae^{-t} + Ate^{-t} - 2Ate^{-t} &= 3e^{-t} \\ -3Ae^{-t} &= 3e^{-t} \\ A &= -1 \\ Y(t) &= -te^{-t} \end{aligned}$$

$$y(t) = c_1 e^{2t} + c_2 e^{-t} - te^{-t}$$

(c) Find the form of $y(t)$ if $g(t) = t^3 - 9te^{5t}\sin 2t$. (Do not solve for the coefficients.)

$$Y(t) = At^3 + Bt^2 + Ct + D + (Et + F)e^{5t}\cos 2t + (Gt + H)e^{5t}\sin 2t$$

$$y(t) = c_1 e^{2t} + c_2 e^{-t} + At^3 + Bt^2 + Ct + D + (Et + F)e^{5t}\cos 2t + (Gt + H)e^{5t}\sin 2t$$