Calculus Review

1. Differentiate each of the following.

   (a) $f(x) = \sqrt{3x^5 - 6x + \frac{x}{x+1}}$

$$f'(x) = \frac{1}{2}\left(3x^5 - 6x + \frac{x}{x+1}\right)^{-1/2}\left(15x^4 - 6 + \frac{1}{(x+1)^2}\right)$$

   (b) $s(t) = 6\sin\omega t$ (where ω is constant).

$$6\omega\cos(\omega t)$$

   (c) $y = t^5 e^{4t}$

$$4t^5 e^{4t} + 5t^4 e^{4t}$$

   (d) $f(x) = \ln(2x+7)$

$$\frac{2}{2x+7}$$

2. Integrate each of the following. All of these use only the basic rules of integration studied in Calculus I.

   (a) $\int x^3 + 5e^{2x}\ dx$

$$\frac{1}{4}x^4 + \frac{5}{2}e^{2x} + C$$

   (b) $\int \sin(2x)\ dx$

$$-\frac{1}{2}\cos(2x) + C$$

   (c) $\int \sec^2 t\ dt$

$$\tan x + C$$

   (d) $\int 3x\cos(4x^2)\ dx$

$$\frac{3}{8}\sin\left(4x^2\right) + C$$

3. Integrate each of the following. For some of these you will have to use the more advanced integration techniques studied in Calculus II.

(a) $\int \frac{dx}{3x-4}$

$$\frac{1}{3}\ln|3x-4|+C$$

(b) $\int t^2 e^{3t}\ dt$

Diff	Anti-Diff
t^2	e^{3t}
$2t$	$\frac{1}{3}e^{3t}$
2	$\frac{1}{9}e^{3t}$
0	$\frac{1}{27}e^{3t}$

$$\frac{1}{3}t^2e^{3t}-\frac{2}{9}te^{3t}+\frac{2}{27}e^{3t}+C$$

(c) $\int \frac{dx}{\sqrt{1-9x^2}}$

$$\begin{aligned} 3x &= \sin\theta \\ dx &= \frac{1}{3}\cos\theta d\theta \end{aligned}$$

$$\begin{aligned} \int \frac{dx}{\sqrt{1-9x^2}} &= \int \frac{\frac{1}{3}\cos\theta d\theta}{\cos\theta} = \int \frac{1}{3}d\theta = \frac{1}{3}\theta + C \\ &= \frac{1}{3}\arcsin(3x)+C \end{aligned}$$

(d) $\int \frac{x-1}{x^2+3x+2}\ dx$

$$\begin{aligned} \frac{x-1}{x^2+3x+2} &= \frac{x-1}{(x+2)(x+1)} = \frac{A}{x+2}+\frac{B}{x+1} \\ x-1 &= A(x+1)+B(x+2) \end{aligned}$$

$$x=-1: \qquad -2=B$$

$$x=-2: \qquad -3=-A$$

$$\begin{aligned} \int \frac{x-1}{x^2+3x+2}\ dx &= \int\left(\frac{3}{x+2}-\frac{2}{x+1}\right)\ dx \\ &= 3\ln|x+2|-2\ln|x+1|+C \end{aligned}$$

(e) $\int \ln(4x)\ dx$

$$\begin{aligned} \int \ln(4x)\ dx &= \int(\ln 4+\ln x)\ dx \\ &= x\ln 4 + x\ln x - x + C \\ &= x\ln(4x) - x + C \end{aligned}$$

4. Determine the radius of convergence and interval of convergence of each of the following series.

(a) $\sum_{n=0}^{\infty} \frac{7^n}{n+2} (x-3)^n$

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \left| \frac{\frac{7^{n+1}}{n+3} (x-3)^{n+1}}{\frac{7^n}{n+2} (x-3)^n} \right| = 7\,|x-3| \lim_{n\to\infty} \frac{n+2}{n+3} = 7\,|x-3|$$

$$\begin{aligned} 7\,|x-3| &< 1 \\ |x-3| &< \frac{1}{7} \\ -\frac{1}{7} &< x-3 < \frac{1}{7} \\ \frac{20}{7} &< x < \frac{22}{7} \end{aligned}$$

(b) $\sum_{n=0}^{\infty} \frac{(n+1)^2 x^n}{n!}$

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \left| \frac{\frac{(n+2)^2 x^{n+1}}{(n+1)!}}{\frac{(n+1)^2 x^n}{n!}} \right| = |x| \lim_{n\to\infty} \frac{(n+2)^2}{(n+1)^3} = 0$$

$$-\infty < x < \infty$$