

Problem #2.107

This problem is definitely too long and convoluted to be a quiz/test problem. But, the individual steps and ideas are all fair game. So make sure you understand each step and how it works. Pretty much every step is based on Theorems 2.12, Theorem 2.13 (which is equivalent to Definition 2.9), and/or Theorem 2.16.

1. (a) Here, we want to repeatedly use: $P(X \cap Y) = P(Y) P(X | Y)$

$$\begin{aligned} P(A \cap B \cap C) &= P(A \cap B) P(C | A \cap B) \\ &= P(A) P(B | A) P(C | A \cap B) \\ &= (0.3)(0.75)(0.2) \\ &= 0.045 \end{aligned}$$

- (b) The following works because A and A' form a partition of S (that is, they are mutually exclusive and contain all possibilities).

$$\begin{aligned} P(B' \cap C) &= P(C \cap B' \cap A) + P(C \cap B' \cap A') \\ &= P(B' \cap A) P(C | B' \cap A) + P(B' \cap A') P(C | B' \cap A') \\ &= P(A) P(B' | A) P(C | B' \cap A) + P(A') P(B' | A') P(C | B' \cap A') \\ &= P(A)(1 - P(B | A)) P(C | B' \cap A) + (1 - P(A))(1 - P(B | A')) P(C | B' \cap A') \\ &= (0.3)(1 - 0.75)(0.8) + (1 - 0.3)(1 - 0.2)(0.9) \\ &= 0.564 \end{aligned}$$

- (c) Here we partition S into the four sets $A \cap B, A \cap B', A' \cap B,$ and $A' \cap B'$.

$$\begin{aligned} P(C) &= P(C \cap A \cap B) + P(C \cap A' \cap B) + P(C \cap A \cap B') + P(C \cap A' \cap B') \\ &= 0.045 + P(A' \cap B) P(C | A' \cap B) + P(A \cap B') P(C | A \cap B') + P(A' \cap B') P(C | A' \cap B') \\ &= 0.045 + P(A') P(B | A') P(C | A' \cap B) + \\ &\quad P(A) P(B' | A) P(C | A \cap B') + P(A') P(B' | A') P(C | A' \cap B') \\ &= 0.045 + (1 - 0.3)(0.2)(0.15) + (0.3)(1 - 0.75)(0.8) + (1 - 0.3)(1 - 0.2)(0.9) \\ &= 0.63 \end{aligned}$$

- (d)

$$\begin{aligned} P(A | C \cap B') &= \frac{P(A \cap C \cap B')}{P(C \cap B')} \\ &= \frac{(0.3)(1 - 0.75)(0.8)}{0.564} \quad \leftarrow \text{the third term in part (c)} \\ &\quad \leftarrow \text{the answer to part (a)} \\ &= 0.10638 \end{aligned}$$