1. TRUE-FALSE. Write TRUE or FALSE (not T or F) as appropriate.

(a) If there are 20 items, there are more ways possible to select an unordered group of 5 of these items than to make a ranked list of 5 of these items.

FALSE. ${}_{20}C_5$ is less than ${}_{20}P_5$.
To understand why, consider that as unordered groups, $\{2, 5, 7, 8, 9\}$ and $\{5, 7, 8, 2, 9\}$ would be counted as the same single group, but as ordered lists, they would be counted separately as two different lists.

(b) If your score on an exam is the 60th percentile then you did better than the median score.

TRUE. You did better than 60% of the people.
Scoring the median score means you did better (and worse) than 50% of the people.

(c) The salaries of American workers would likely be normally distributed.

FALSE. The median salary is around $30,000 (according to the US Bureau of Labor and Statistics in 2002).
The highest paid workers make well into the millions. So the distribution of salaries would have a much longer tail to the right than it would on the left (since salaries can't go lower than $0).

(d) For a normal distribution, the mean, the median, and the mode are all the same.

TRUE.

2. TRUE-FALSE. Write TRUE or FALSE (not T or F) as appropriate.

(a) It is possible to have a probability larger than 1.

FALSE.

(b) It is possible for the mean of a data set to be larger than the median.

TRUE. For example $\{1, 2, 9\}$

(c) It is possible for a z-score to be negative.

TRUE. Any data value that is less than the median will correspond to a negative z-score.

(d) If 40% of all men support a particular candidate, and 40% of all women also support that candidate, then that candidate has the support of 80% of the people.

FALSE. Even if there were identical numbers of men and women the candidate would only have the support of 40% of the people, not 80%.

3. If a person randomly picks 2 cards from a deck (without replacement), what is the probability that both cards are hearts?

$$P\left(1^{st} \text{ card is } \heartsuit\right) \cdot P\left(2^{nd} \text{ card is } \heartsuit\right) = \frac{13}{52} \cdot \frac{12}{51} = \frac{156}{2652} = 0.0588 = 5.88\%$$

4. If a person randomly picks 2 cards from a deck (without replacement), what is the probability that at least one of them is an Ace?

First we will figure out the probability that neither is an Ace.

$$P\left(1^{st} \text{ card is not an Ace}\right) \cdot P\left(2^{nd} \text{ card is not an Ace}\right) = \frac{48}{52} \cdot \frac{47}{51} = \frac{2256}{2652} = 0.8507 = 85.07\%$$

Then, we calculate as follows:

$$\begin{aligned} P\left(\text{At least one is an Ace}\right) &= 1 - P\left(\text{neither is an Ace}\right) \\ &= 1 - 0.8507 \\ &= 0.1493 \\ &= 14.93\% \end{aligned}$$

5. A company randomly assigns computer passwords to its employees. The passwords are all 9 characters long, made up of 3 letters (a - z) followed by 5 digits (0 - 9) followed by a symbol which is either &,#, or $. (for example, ece3692& or zak2222#) <u>How many</u> different computer passwords can this company assign?

$$26 \cdot 26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 3 = 5,272,800,000$$

6. What is the <u>probability</u> that a computer password randomly assigned by the company in question 5 begins with the letter 'g', has all of the digits the same, and ends with a dollar sign? (for example, gzx7777$ or ggg0000$)

$$\frac{1 \cdot 26 \cdot 26 \cdot 10 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1}{26 \cdot 26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 3} = \frac{1}{780,000}$$

OR

$$\frac{1}{26} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{3} = \frac{1}{780,000}$$

7. There are 250 people attending a mental health conference. 100 of them are psychiatrists, 90 of them are psychologists, and 60 of them are social workers. (There is no overlap among the groups.) The conference organizers wish to create a discussion panel comprised of 5 psychiatrists, 4 psychologists, and 4 social workers. In how many ways can this be done?

$$\begin{aligned} {}_{100}C_5 \cdot {}_{90}C_4 \cdot {}_{60}C_4 &= 9.380825562 \cdot 10^{19} \\ &= 93,808,255,620,000,000,000 \end{aligned}$$

YOWZA! That's a lot of possibilities.

8. Kim and Stan are two different people standing in a line of 50 people waiting to buy concert tickets. A local radio station announces that they will give free tickets to the person that is 15th in line and a free t-shirt to the person that is 30th in line. What is the probability that Kim will get the free tickets and Stan will get the free t-shirt?

$$P\left(\text{Kim is } 15^{th} \text{ in line}\right) \cdot P\left(\text{Stan is } 30^{th} \text{ in line}\right) = \frac{1}{50} \cdot \frac{1}{49} = 0.000408 = 0.0408\%$$

OR

$$\frac{{}_{48}P_{48}}{{}_{50}P_{50}} = 0.0408\%$$

9. Suppose that the following data is collected from a survey of 800 college students:

	Has chosen a major	Has not chosen a major
Freshman	90	110
Sophomore	100	100
Junior	150	50
Senior	190	10

What is the probability that a randomly chosen student in this survey has not chosen a major?

$$P\left(\text{not chosen a major}\right) = \frac{110 + 100 + 50 + 10}{800} = \frac{270}{800} = 0.3375 = 33.75\%$$

10. What is the probability that a randomly chosen student in the survey described in question 9 is a Junior or a Senior given that the student has not chosen a major?

$$P\left(\text{Jr. or Sr.} \mid \text{not chosen a major}\right) = \frac{50 + 10}{270} = \frac{60}{270} = 0.2222 = 22.22\%$$

11. You have a chance to get in on a stock deal that you can buy into for $1000. After a careful analysis, you estimate that your chance of the stock going up is 40%, and the chance of the stock going down is 25%. Further, you estimate that your original $1000 of stock will be worth $10,000 if the stock goes up and will be worth $0 if the stock goes down (and will be worth $1000 if the stock stays the same). Based on these estimates, what is your expected value if you enter into this stock deal?

Outcome	Gain/Loss	Probability	(G/L)*Prob
Stock up	$9,000	0.40	$3,600
Stock down	-$1,000	0.25	-$250
No change	$0	0.35	0
		Sum:	$3,350

The expected value is $3,350.

12. Consider the following frequency distribution

Value	Frequency
10	7
20	4
30	2
40	2
50	1
60	6

Calculate the mean of this data set.

$$\frac{10*7+20*4+30*2+40*2+50*1+60*6}{7+4+2+2+1+6} = \frac{700}{22} = 31.82$$

13. Calculate the median of the data set in question 12.

There are 22 data items. So the median is the average of the 11^{th} and the 12^{th} ordered item.

$$\text{median} = \frac{20+30}{2} = 25$$

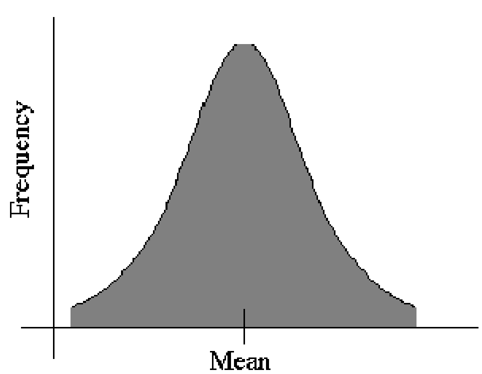
14. The mean of the following data set is 4. Calculate the standard deviation.

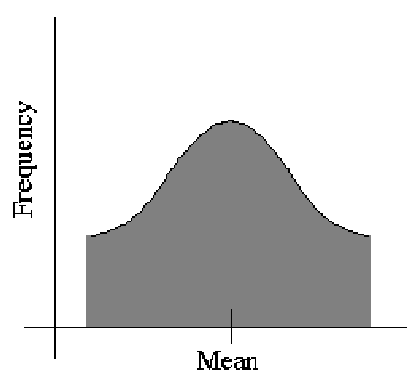
$$1, 2, 3, 3, 6, 9$$

value	deviation	deviation squared
1	-3	9
2	-2	4
3	-1	1
3	-1	1
6	2	4
9	5	25
	sum:	44

$$\text{Standard Deviation} = \sqrt{\frac{44}{6-1}} = \sqrt{\frac{44}{5}} = \sqrt{8.8} = 2.966$$

15. Which of the following two distributions has the largest standard deviation? (Assume the scaling on the axes is the same in both graphs.)





The distribution on the right has the largest standard deviation. More of the data falls farther away (i.e., deviates more) from the mean than it does in the distribution on the left.

16. Assume the birth weight of kittens is normally distributed with a mean of 3 ounces and a standard deviation of 0.1 ounces. What percentage of kittens weigh between 2.9 ounces and 3.1 ounces at birth?

    These values are 1 standard deviation from the mean on the left and right, respectively. So, by the 68-95-99.7 rule, we see that 68% of newborn kittens weigh between 2.9 ounces and 3.1 ounces. (Or at least that would be the case if I hadn't made up the values in the question.)

17. According to the information given in question 16, what percentage of kittens weigh less than 2.7 ounces at birth?

    2.7 is 3 standard deviations to the left of the mean. Using the 68-95-99.7 rule and the fact that 50% of these kittens weigh less than the mean weight, we get $50\% - \frac{1}{2}(99.7\%) = 0.15\%$ weighing less than 2.7 ounces.

18. Assume that at a particular company, the amount of time that customers are left waiting on hold is normally distributed with a mean of 180 seconds and a standard deviation of 20 seconds. What is the z-score that corresponds to a wait time of 190 seconds?

$$z = \frac{190 - 180}{20} = \frac{10}{20} = 0.5$$

19. For the company in question 18, what percentage of their customers end up waiting on hold for longer than 190 seconds?

    From the z-score table, we see that 69.15% of customers wait on hold for shorter than 190 seconds. Thus, 30.85% of customers wait for longer.