Math 110 Name KEY

Quiz 5

Every answer must be fully justified. Show your work!!! Your answers may include factorials if appropriate, but do not leave expressions like ${}_{200}P_{50}$ or ${}_{200}C_{50}$.

1. Suppose that a 6-sided die is rolled 3 times.

   (a) What is the probability that all three numbers rolled are even?

$$\text{Method 1:}\quad P(\text{all even}) = \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} = \frac{1}{8}$$

$$\begin{aligned}\text{Method 2:}\quad P(\text{all even}) &= \frac{\#\text{ of different ways to get all evens}}{\#\text{ of different ways to roll the dice}}\\ &= \frac{3\cdot 3\cdot 3}{6\cdot 6\cdot 6}\\ &= \frac{1}{8}\end{aligned}$$

   (b) What is the probability that at least one odd number is rolled?

$$\begin{aligned}P(\text{at least one odd}) &= 1 - P(\text{all even})\\ &= 1 - \frac{1}{8}\\ &= \frac{7}{8}\end{aligned}$$

2. For this problem assume that in Illinois license plates have to be made up of 2 letters followed by 4 digits (for example, HF 2335 or AA 0679).

   (a) What is the probability that a randomly chosen license plate begins with A and has no zeroes in it?

$$\text{Method 1:}\quad P(\text{begins with A, no zeroes}) = \frac{1}{26}\cdot\frac{9}{10}\cdot\frac{9}{10}\cdot\frac{9}{10}\cdot\frac{9}{10} = \frac{6561}{260\,000}$$

$$\begin{aligned}\text{Method 2:}\quad P(\text{begins with A, no zeroes}) &= \frac{\#\text{ of different plates of the kind we want}}{\#\text{ of different plates}}\\ &= \frac{1\cdot 26\cdot 9\cdot 9\cdot 9\cdot 9}{26\cdot 26\cdot 10\cdot 10\cdot 10\cdot 10}\\ &= \frac{6561}{260\,000}\end{aligned}$$

   (b) What is the probability that a randomly chosen license plate is JB 6332?

$$\begin{aligned}P(\text{getting exactly the plate JB 6332}) &= \frac{1}{\#\text{ of different plates}}\\ &= \frac{1}{26\cdot 26\cdot 10\cdot 10\cdot 10\cdot 10}\\ &= \frac{1}{260\,000}\end{aligned}$$

3. Suppose that a movie theater has 30 classic movies available for a special All-night Classic Marathon, during which it will show 10 of these classic movies. Casablanca is one of the movies available.

   (a) What is the probability that Casablanca will be shown first during this marathon?

   Method 1: $P(\text{begins with Casablanca}) = \frac{1}{30}$ (since we only care about choosing the first movie)

   Method 2:
   $$P(\text{begins with Casablanca}) = \frac{\#\text{ of different lineups that begin with Casablanca}}{\#\text{ of different lineups}}$$
   $$= \frac{{}_1P_1 \cdot {}_{29}P_9}{{}_{30}P_{10}}$$
   $$= \frac{1}{30}$$

   (b) What is the probability that Casablanca will be shown during this marathon?

   Method 1: Since there is a $\frac{1}{30}$ chance for each of the 10 positions,
   $$P(\text{Casablanca gets shown}) = \underbrace{\frac{1}{30} + \frac{1}{30} + \cdots + \frac{1}{30}}_{\text{10 of these}} = \frac{1}{3}$$

   Method 2:
   $$P(\text{Casablanca doesn't get shown}) = \frac{\#\text{ of different lineups without Casablanca}}{\#\text{ of different lineups}}$$
   $$= \frac{{}_{29}P_{10}}{{}_{30}P_{10}}$$
   $$= \frac{2}{3}$$
   $$P(\text{Casablanca gets shown}) = 1 - P(\text{Casablanca doesn't get shown})$$
   $$= 1 - \frac{2}{3}$$
   $$= \frac{1}{3}$$

4. Suppose the following game is proposed to you: You pay \$1 to play the game (which is <u>not</u> returned to you if you win). You draw a card from a deck of 52 cards. If you draw an Ace you win \$10. If you draw a 10, Jack, Queen or King, you win \$2. If you draw the 2 of clubs, you win \$20. Determine your expected value for this game.

Outcome	Gain/Loss	Probability	Product
Ace	\$9	$\frac{4}{52}$	$\frac{36}{52}$
10, J, Q, or K	\$1	$\frac{16}{52}$	$\frac{16}{52}$
2♣	\$19	$\frac{1}{52}$	$\frac{19}{52}$
Anything Else	-\$1	$\frac{31}{52}$	$-\frac{31}{52}$
			Sum= $\frac{40}{52} = .76923$

So the expected value is 76.9¢.