# The Geometry of the Universe

## (*Some Applications of Non-Euclidean Geometry*)

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Mathematics & Physics Seminar
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# An episodic presentation

(with mentions of observational evidence)

- Euclidean Geometry
- Quick Applications
- Saga of Non-Euclidean Geometry
- Special Relativity
- General Relativity and Gravity
- General Relativity and the Universe
- This Week’s Anomaly

# Euclid of Alexandria ~300 BCE

## *Elements*

- Axiomatic system
- Plane Geometry
- Solid Geometry (3d)
- Number theory
- Algebra (in geometric language)

## Postulates (constructive)

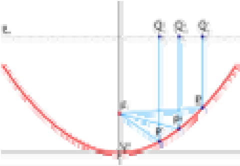
1. To draw a straight line from any point to any point.
2. To produce [extend] a finite straight line continuously in a straight line.
3. To describe a circle with any centre and distance [radius].
4. That all right angles are equal to one another.
5. That, if a straight line falling on two straight lines make the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles.

## Common Notions

1. Things that are equal to the same thing are equal to one another.
2. If equals are added to equals, then the wholes are equal.
3. If equals are subtracted from equals, then the remainders are equal.
4. Things that coincide with one another, are to equal one another.
5. The whole is greater than the part.

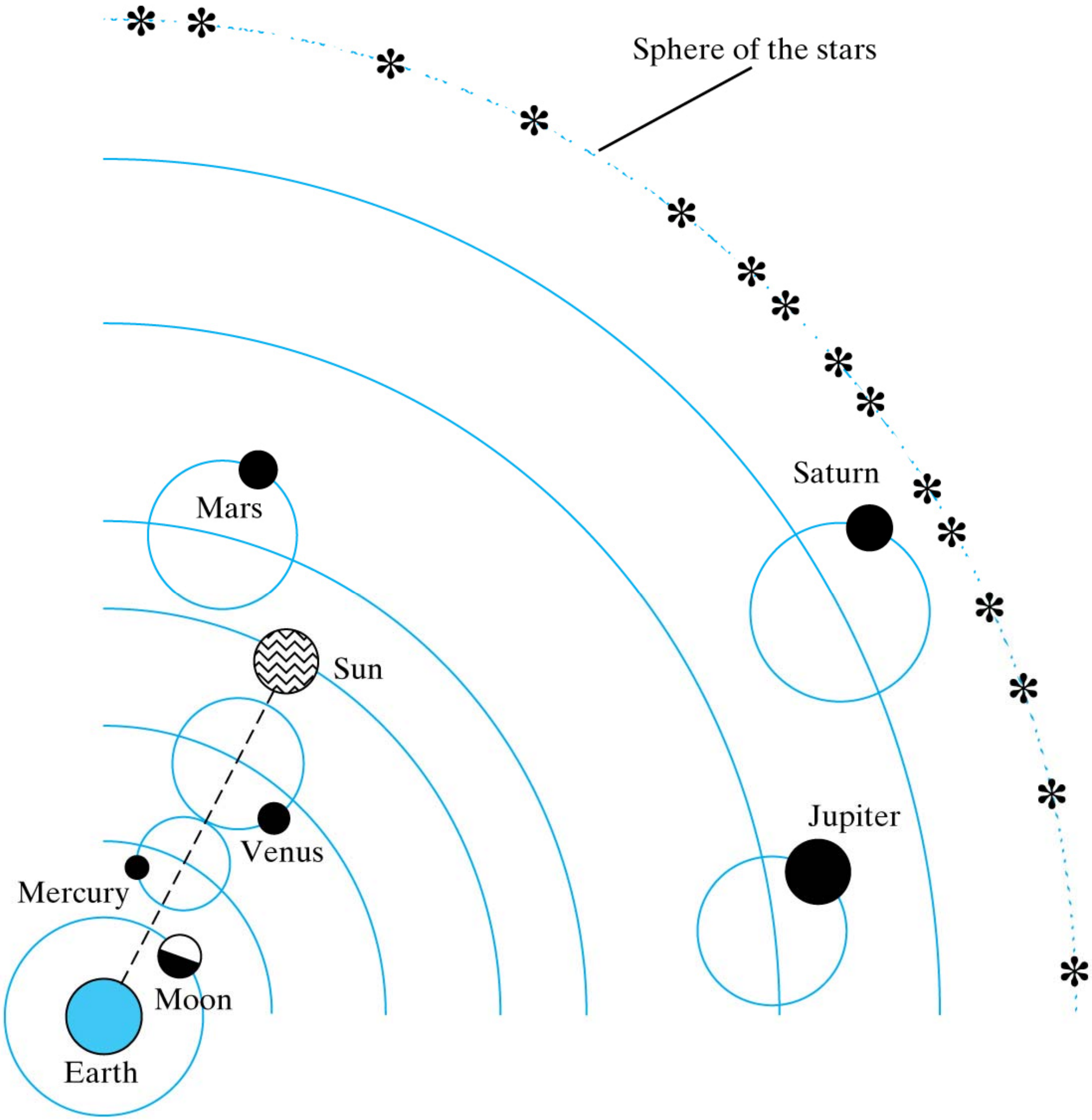
A surveyor uses a Level

Sphere packing applies to a stack of oranges.



A parabolic mirror brings parallel rays of light to a focus.

# Figure 1.11

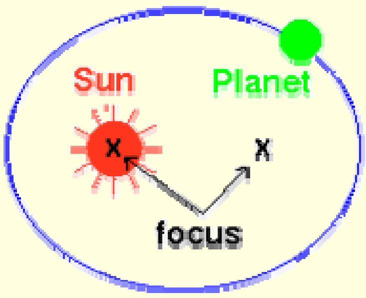

Sphere of the stars
Saturn
Mars
Sun
Jupiter
Venus
Mercury
Moon
Earth

# The Laws of Planetary Motion

**Kepler obtained Brahe's data after his death despite the attempts by Brahe's family to keep the data from him in the hope of monetary gain. There is some evidence that Kepler obtained the data by less than legal means; it is fortunate for the development of modern astronomy that he was successful. Utilizing the voluminous and precise data of Brahe, Kepler was eventually able to build on the realization that the orbits of the planets were ellipses to formulate his *Three Laws of Planetary Motion*.**

## Kepler's First Law:

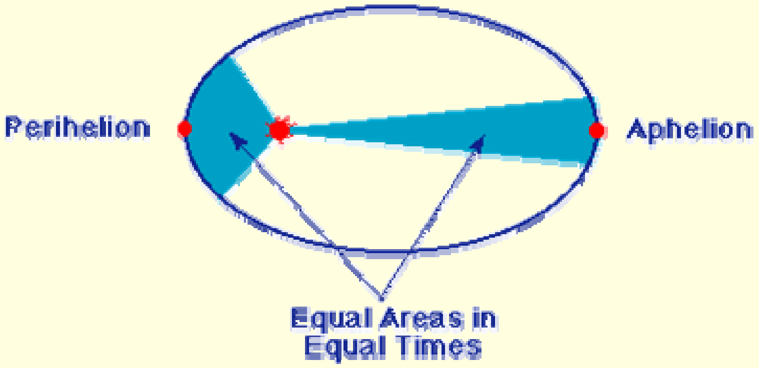
**I. The orbits of the planets are ellipses, with the Sun at one focus of the ellipse.**



**Kepler's First Law is illustrated in the image shown above. The Sun is not at the center of the ellipse, but is instead at one focus (generally there is nothing at the other focus of the ellipse). The planet then follows the ellipse in its orbit, which means that the Earth-Sun distance is constantly changing as the planet goes around its orbit. For purpose of illustration we have shown the orbit as rather eccentric; remember that the actual orbits are much less eccentric than this.**

## Kepler's Second Law:

**II. The line joining the planet to the Sun sweeps out equal areas in equal times as the planet travels around the ellipse.**



**Proclus (410-485)** (Commentary and false proof)
**Omar Khayyam (1048-1131)**
**Saccheri (1667-1773)** (False proof – contradiction)
**Kant: Euclidean geometry is "the inevitable necessity of thought."**
**Legendre (1752-1833) 1794** (40 years)

***The sum of the angles of a triangle is equal to two right angles.***

**Playfair (1748-1819) 1795**

***Given a line and a point not on the line, it is possible to draw exactly one line through the given point parallel to the line.***

**Gauss (1777-1855) (Didn't publish)**
**Janos Bolyai (1802-1860) 1823** (Anticipated by Gauss)
**Lobachevsky (1792-1856) (pub. 1829!)**

(considered 5th postulate as a special case)

**Riemann (1826- 1866)**

(spherical geometry, no parallel lines -- **1854**)

**Beltrami (1835-1900) 1868**
**Felix Klein (1849-1925) 1871**
**David Hilbert (1862-1943) 1899**

(General relativity field equations – 1915/1916)

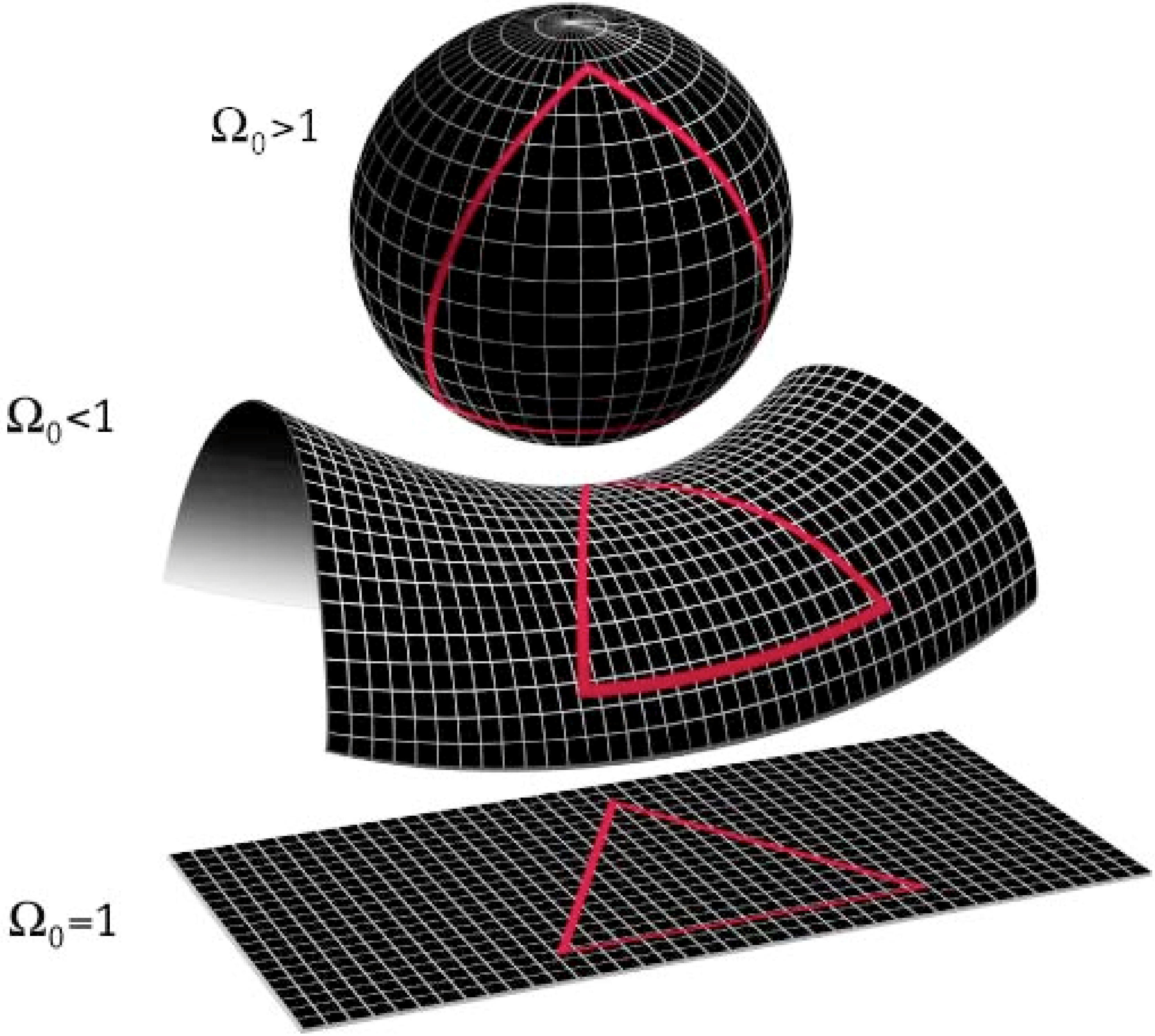
**Jules Henri Poincare (1854-1912)** (algebraic topology)

(Almost special relativity)

**Hermann Minkowski (1864-1909) 1907**

(Space-time geometry for special relativity)

| Type of geometry | Number of parallels | Sum of angles in a triangle | Ratio of circumference to diameter of circle | Measure of curvature |
|---|---|---|---|---|
| **Hyperbolic** | Infinite | $< 180^{o}$ | $> \pi$ | $< 0$ |
| **Euclidean** | 1 | $180^{o}$ | $\pi$ | 0 |
| **Elliptical** | 0 | $> 180^{o}$ | $< \pi$ | $> 0$ |



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# Albert Einstein (1879-1955)

## Special Relativity – 1905

*On the Electrodynamics of Moving Bodies*

George FitzGerald
Hendrik Lorentz
Henri Poincaré
Hermann Minkowski (later)

# Conceptual Foundations of Classical Physics and Special Relativity

| **Galileo/Newton (Classical Physics)** | **Einstein (Special Relativity)** |
|---|---|
| Space is absolute.<br>Time is absolute.<br>**Laws of Physics are absolute.** | **Laws of Physics are absolute**.<br>**Speed of Light in vacuum is absolute (c = c)** |
| **<u>All</u>** velocities are relative. (They combine by simple vector addition: just +/- signs in one dimension.) | Space is relative. ("Moving objects are shortened along their direction of motion.")<br>Time is relative. ("Moving clocks run slower.")<br>Velocities other than c are relative. (They combine by a revised formula that ensures c = c. It is still necessary to keep track of directions.) |

**Notes:**

1. Here "absolute" means "the same in all non-accelerating reference frames" or equivalently, "invariant".
2. Here "relative" means "dependent on the motion of the (non-accelerating) reference frame of the observer".
3. Here (and many places) "c" is used as a symbol for the speed of light in vacuum.

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$ds^2 = dx^2 + dy^2 + dz^2 – c^2dt^2$

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## General Relativity – 1907-1915

Accelerating frames → Theory of Gravity
Geometry of Riemann…

David Hilbert

http://www.youtube.com/watch?v=O-p8yZYxNGc

# Albert Einstein (1879-1955)

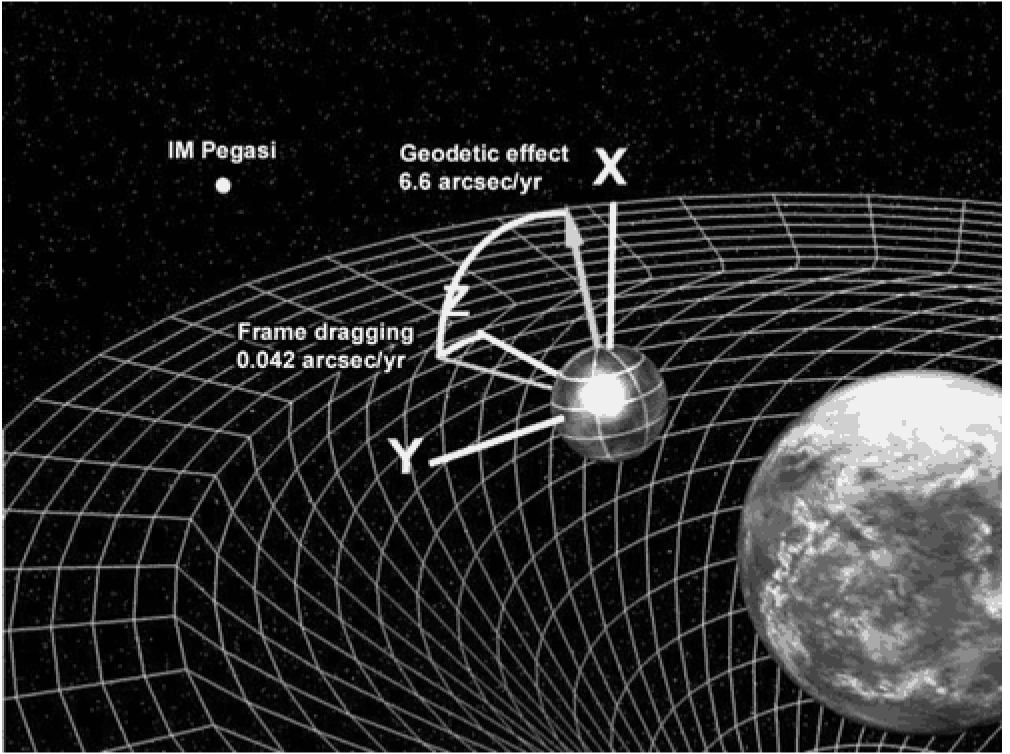
## General Relativity – 1907-1915

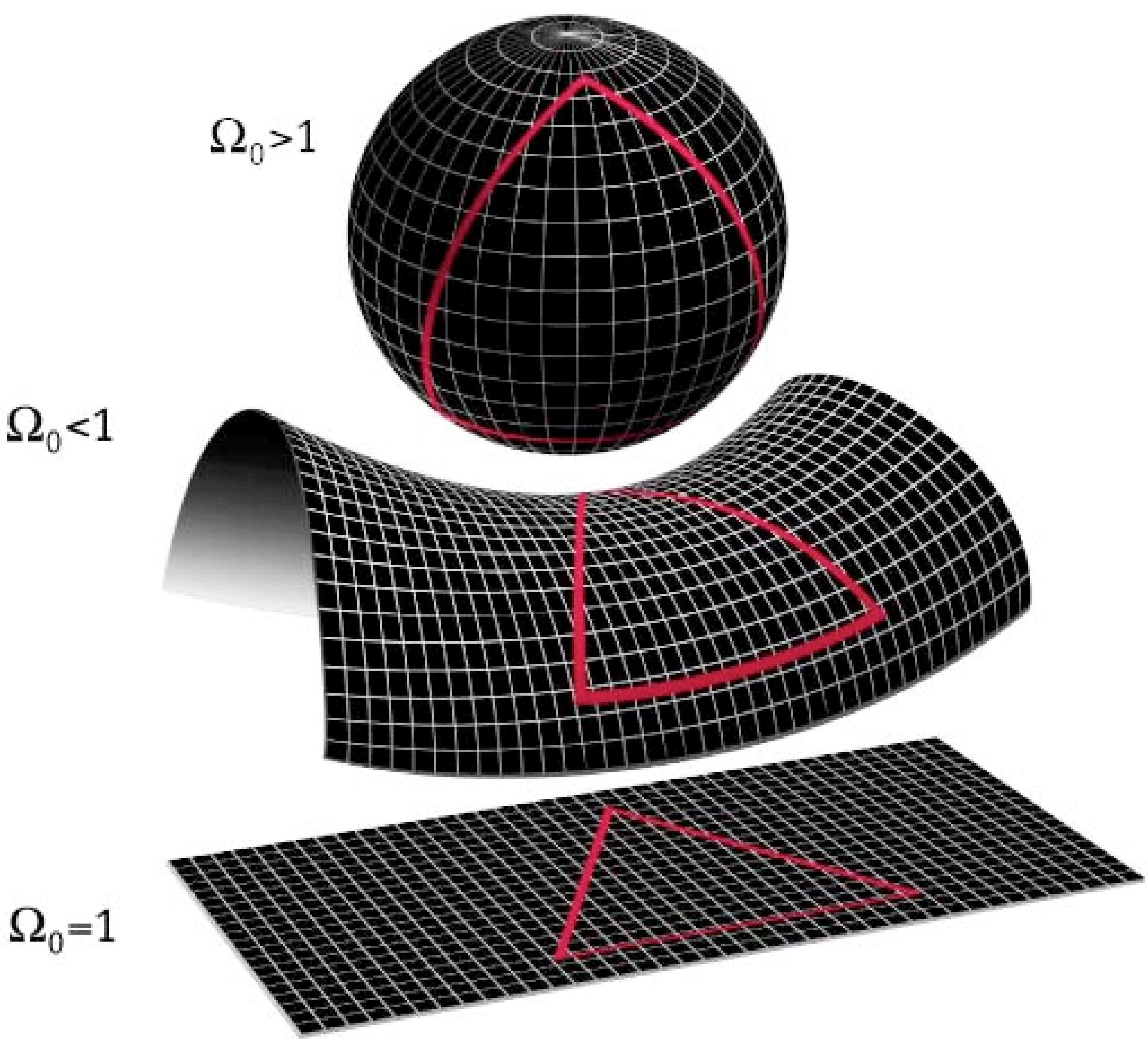
Accelerating frames → Theory of Gravity
Geometry of Riemann…
David Hilbert

## Tests

Precession of Mercury’s Perihelion
(43"/century anomaly)
Bending of starlight (1.75' of arc)
Gravitational redshift
Gravitational lensing
Light travel time delay
Twin paradox with atomic clocks
The GPS
Gravity Probe B
Frame-dragging & the geodetic effects
Gravitational waves (searching)

……


IM Pegasi
Geodetic effect
6.6 arcsec/yr
X
Frame dragging
0.042 arcsec/yr
Y



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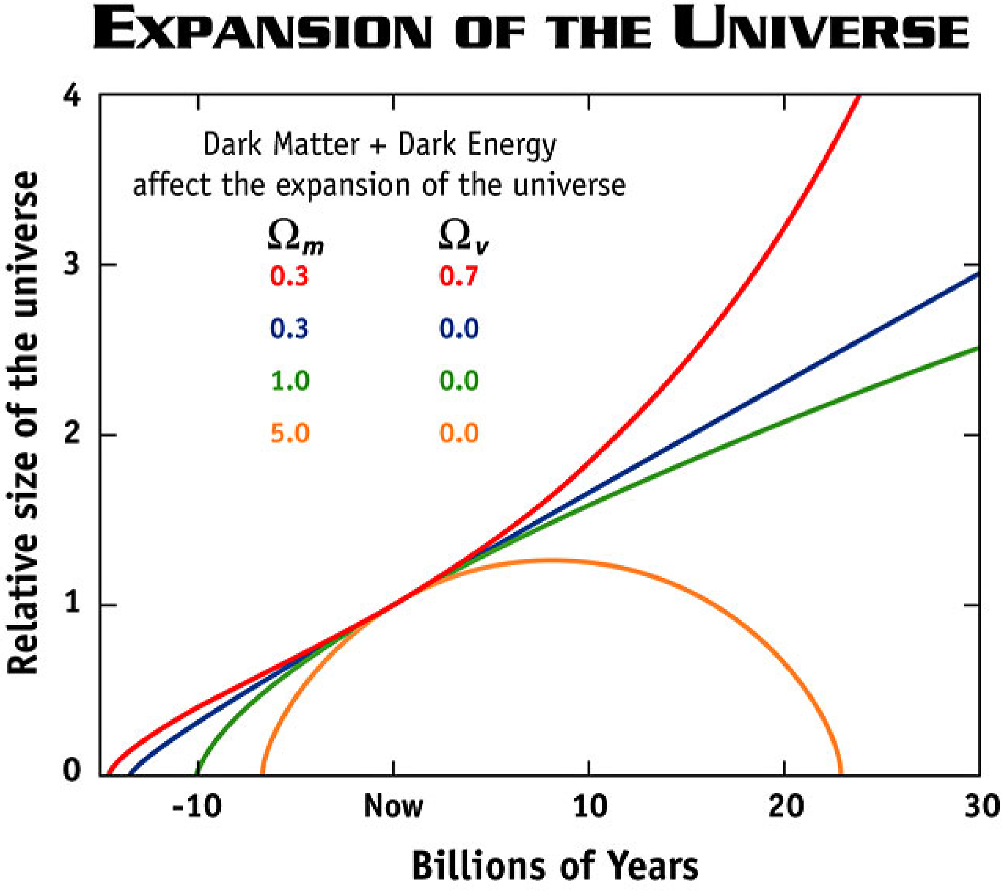
# Expansion of the Universe

### Long Description (Return to Concepts page)

Possible scenarios for the expansion (and possibly contraction) of the universe: the bottom orange curve represents a closed, high density universe which expands for several billion years, then ultimately turns around and collapses under its own weight. The green curve represents a flat, critical density universe in which the expansion rate continually slows down (the curves becomes ever more horizontal). The blue curve shows an open, low density universe whose expansion is also slowing down, but not as much as the previous two because the pull of gravity is not as strong. The top (red) curve shows a universe in which a large fraction of the matter is in a form dubbed "dark energy" which is causing the expansion of the universe to speed up (accelerate). There is growing evidence that our universe is following the red curve.



# Measurement of the neutrino velocity with the OPERA detector in the CNGS beam

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# Abstract

The OPERA neutrino experiment at the underground Gran Sasso Laboratory has measured the velocity of neutrinos from the CERN CNGS beam over a baseline of about 730 km with much higher accuracy than previous studies conducted with accelerator neutrinos. The measurement is based on high-statistics data taken by OPERA in the years 2009, 2010 and 2011. Dedicated upgrades of the CNGS timing system and of the OPERA detector, as well as a high precision geodesy campaign for the measurement of the neutrino baseline, allowed reaching comparable systematic and statistical accuracies. An early arrival time of CNGS muon neutrinos with respect to the one computed assuming the speed of light in vacuum of (60.7 ± 6.9 (stat.) ± 7.4 (sys.)) ns was measured. This anomaly corresponds to a relative difference of the muon neutrino velocity with respect to the speed of light $(v-c)/c = (2.48 \pm 0.28 \text{ (stat.)} \pm 0.30 \text{ (sys.)}) \times 10^{-5}$.

## 1. Introduction

The OPERA neutrino experiment [1] at the underground Gran Sasso Laboratory (LNGS) was designed to perform the first detection of neutrino oscillations in direct appearance mode in the $\nu_\mu \rightarrow \nu_\tau$ channel, the signature being the identification of the $\tau^-$ lepton created by its charged current (CC) interaction [2].

In addition to its main goal, the experiment is well suited to determine the neutrino velocity with high accuracy through the measurement of the time of flight and the distance between the source of the CNGS neutrino beam at CERN (CERN Neutrino beam to Gran Sasso) [3] and the OPERA detector at LNGS. For CNGS neutrino energies, $\langle E_\nu \rangle = 17$ GeV, the relative deviation from the speed of light $c$ of the neutrino velocity due to its finite rest mass is expected to be smaller than $10^{-19}$, even assuming the mass of the heaviest neutrino *eigenstate* to be as large as 2 eV [4]. Hence, a larger deviation of the neutrino velocity from $c$ would be a striking result pointing to new physics in the neutrino sector. So far, no established deviation has been observed by any experiment.

In the past, a high energy ($E_\nu > 30$ GeV) and short baseline experiment has been able to test deviations down to $|v-c|/c < 4\times10^{-5}$ [5]. With a baseline analogous to that of OPERA but at lower neutrino energies ($E_\nu$ peaking at ~3 GeV with a tail extending above 100 GeV), the MINOS experiment reported a measurement of $(v-c)/c = 5.1 \pm 2.9\times10^{-5}$ [6]. At much lower energy, in the 10 MeV range, a stringent limit of $|v-c|/c < 2\times10^{-9}$ was set by the observation of (anti) neutrinos emitted by the SN1987A supernova [7].

In this paper we report on the precision determination of the neutrino velocity, defined as the ratio of the precisely measured distance from CERN to OPERA to the time of flight of neutrinos travelling through the Earth's crust. We used the high-statistics data taken by OPERA in the years 2009, 2010 and 2011. Dedicated upgrades of the timing systems for the time tagging of the CNGS beam at CERN and of the OPERA detector at LNGS resulted in a reduction of the systematic uncertainties down to the level of the statistical error. The measurement also relies on a high-accuracy geodesy campaign that allowed measuring the 730 km CNGS baseline with a precision of 20 cm.