Investigating the effect of periodic forcing on localized traveling waves coupled to an additional mode

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Abstract. I will investigate how localized structures respond to periodically turning on a parameter in the system. A computer workstation will also be set up to perform the numerical computations.

I. Project Summary

Pattern formation is the study of periodic or ordered patterns found in different systems. Examples of patterns observed in nature include sand ripples, zebra stripes, and fingerprints. Patterns also appear in many experimental systems. Of particular interest are observations of a small, isolated region containing a patterned state that is embedded within a non-patterned state. These states are generally called localized structures. This proposed project will investigate such localized states using both mathematical techniques and numerical computations. The investigation is particularly aimed at trying to explain why localized states, called “worms,” emerge in a particular experimental system. A mathematical model has been proposed to explain the worms, but a detailed investigation is still needed to test the validity of the model. This project will provide the mathematical understanding for testing the model and ultimately comparing results with experimental tests.

The proposed project consists of using mathematical and numerical techniques to investigate the model equations and the effect of changing various parameters. Time will also be devoted to setting up a computational workstation and modifying existing programs to solve the equations proposed as a model. Once these codes are functioning, they can be modified to investigate other equations. Hence, this project is beneficial in providing the computational foundation for my future research investigations. Moreover, projects of this computational nature are well-suited to undergraduate research. Students can compare the results of the computations with the analytical results without needing the advanced knowledge of computer programming.

II. Narrative

Current Situation

In addition to patterns found naturally, many experimental systems can also form patterns when they are forced out of their equilibrium state. Rayleigh-Bènard convection, in which a thin fluid layer is heated from below, is a classic example. Initially, the fluid conducts the warmer fluid up to the cooler surface. Above a critical temperature, a periodic roll structure emerges in the now convecting fluid. Experimental and theoretical investigations of patterns are aimed at understanding the mechanisms involved in the formation and selection of patterns as well as their stability and how they change over time. Equations describing the underlying microscopic dynamics of such systems are typically quite complicated. Hence, one attempts to describe the behavior and patterns on a large scale with simpler model equations. Current understanding of patterns that fill the entire space is well advanced. In the continuing study of pattern formation, much current work focuses on more complex structures such as the localized states considered in this proposal. A localized structure is one in which the pattern is confined to a small spatial region and surrounded by a non-patterned state.

Observations of localized structures in experimental systems have led to additional questions in pattern formation. One question of particular interest is why such states do not expand to fill the entire system. Hence, one is interested in understanding the process by which they come into existence and when they are stable. Once again, these localized structures are often studied within simpler model equations, not only due to the complexity of underlying microscopic equations (if even known), but also to provide a universality for describing similar states in diverse systems. The implications of localized structures in any given system may vary from being highly disruptive and unwanted to being of great import such as solitons with applications in nonlinear optics associated with communication networks. Hence, a thorough understanding of localized structures is needed in order to successfully suppress or exploit such states.

This proposal addresses localized structures and their localization mechanisms as motivated by experimental observations. First, in convection experiments where a thin layer of a water and alcohol mixture is heated from below, localized pulses emerge that travel through the system. The heating of the fluid causes the concentration of alcohol in the mixture to slowly decrease. This slow decay of the alcohol concentration is a key factor in explaining the stable existence of these pulses as well as explaining some of their attributes, such as their speed. Another observation has been in electroconvection of nematic liquid crystals, where a thin layer of liquid crystals is places between to plates and an electric voltage is applied. Localized waves, called “worms” due to their appearance, have been observed for certain values of the voltage.

Although the pulses in the alcohol-water experiments are well understood, the reasons for the worm states observed in electroconvection are still being investigated. However, some of their behavior and characteristics are consistent with results for the pulses. In particular, it has been suggested that there is a component in electroconvection that decays slowly. Motivated by the similarity to the slowly decaying concentration of alcohol in the alcohol-water convection and the resulting pulses, a similar mathematical model was suggested to describe the worm states. This model seems to capture the essence of the worms observed, but further investigation is required to determine whether a slowly decaying component is actually relevant in the experiments. Furthermore, research is still being done to determine which component is, in fact, the one decaying slowly.

One way to test the validity of the model is to obtain the pulse or worm states and then change an externally controlled parameter in the system to see how the pulses respond. One example would be to periodically add a burst of heat to the fluid experiments or periodically increase the voltage in the electroconvection. This periodic forcing can also be accounted for in the mathematical equations suggested as a model. Then, by studying the effect of changing the value of the parameter in the equations, one can observe the effect on a pulse state in our model equations. In particular, one can observe the effect when a slowly decaying component is present and when it is not. The results of the mathematical investigation will aid researchers in knowing what measurements to take in the experiments as well as possibly designing new experiments to test the conclusions of the mathematical investigation.

The Project Plan

The primary goals of this project are two-fold. First, I will investigate how introducing a periodically forced parameter affects already existing localized structures using mathematical and numerical methods. The results should be relevant for both pulses in (alcohol-water) fluid convection and for worms in electroconvection. The second goal is to set-up a computational workstation for my research. Once the workstation and my codes are functioning, it not only obviously aids my research, but greatly broadens my ability to include undergraduate students in these investigations.

In terms of the first goal, the project will progress in stages. The equations of interest have already been suggested as a model to describe the observed behavior. Initially the research conducted this summer will be to analyze these equations using various mathematical techniques. These "hand" calculations are used to make predictions about expected behavior as parameters in the equations are changed. Understanding which terms in the equations lead to certain changes in behavior helps in understanding the underlying physical processes causing the changes. The equations are quite complex and analytical techniques are limited in scope. Hence, the second phase of the research focuses on numerically solving the equations. Using results of the analytical investigation, I can confirm their validity by observing the expected behavior in the numerical solutions. Once the analytical results are confirmed and understood, I can use the numerical computations to investigate further behavior and draw conclusions that were not accessible in the purely analytical research. Ultimately, the third stage of the research will be to communicate the results with experimentalists. Based on the analytical and numerical investigation, we can make predictions about the behavior expected in the experiments. Hence, they can test whether the proposed model truly explains the necessary factors needed for the localized states. These stages of research all feedback into each other as the model is refined and modified and new questions are addressed, based on whether the resulting numerics and experiments either confirm or contradict the expected results.

The intense numerical computations needed for this project leads to the second primary goal of establishing a computational workstation. I already have several existing FORTRAN programs containing thousands of lines of code that can be modified and used to solve the equations of interest. Once a workstation is obtained, time will be spent this summer testing and debugging the code on the new workstation to verify that the code works properly and can be used to solve the equations. For one of the programs, it is anticipated that this process will be relatively simple since the code was successfully running on Unix and Linux operating systems I used in graduate school. The other main program has been run only on a Unix machine and may require more work to convert it to a Linux system. The similarities in the operating systems and compiler options make this a reasonable task. Once the programs are functional, they can be used for the numerical investigation in this project, as well as for other future projects.

Since coming to Elmhurst College, I have concentrated my research efforts on writing up results and in analytical calculations. At this stage, it is now important to begin investigating the problems numerically. The initial start-up time required to set up the workstation and the programs is well worth it, as I should be able to use them for years with very little maintenance. Additionally, undergraduate students can then use the working programs to help me in my research. The programs have menus that allow the user to change parameters and it has a graphical output so that one can see the results. Although the actual analytical calculations are quite advanced for a typical undergraduate student, the results of the analysis can often be understood with concepts from our differential equations and calculus courses. Hence, students should be able to interpret the results of a numerical solution and compare it with the analytical study. It is important to me, as well as a focus of the department and college, to provide students with this kind of opportunity to be directly involved in research. Without the students able to do numerical computations the depth of research with which I can involve them is very limited.

Although not part of this proposed project, I have analytical results from a project with a student, Ben Blaiszik, which he presented at the Mathematical Association of America Annual meeting last summer. By setting up the workstation and codes, I will immediately have the ability to continue that project with another student.

Faculty Expertise

My general research area is in the formation of patterns in nonlinear dynamical systems. My graduate work focused specifically on localized (i.e. isolated pulse-type) structures in such systems. My goal is to understand and explain the processes by which patterns and localized states emerge in physical systems and to explain mathematically the mechanisms responsible for stabilizing the localized state. Through results of my graduate investigations, I was able to provide an explanation for the existence of pulse-type structures called oscillons in experiments where layers of granular materials (e.g. sand, beads, spheres, etc.) are shaken vertically. More relevant to this proposed project was the research I performed investigating localized pulses in traveling waves. We discovered that periodically forcing a parameter in the system results in a *new* process by which a localized pulse may emerge. That project provides the foundation for investigating further the effect of periodically forcing a system with already existing localized structures and observing how they respond to the forcing.

Through my graduate work, I developed the mathematical skills in nonlinear and asymptotic analysis, bifurcation theory, dynamical systems, and mathematical modeling necessary for approaching these problems analytically. I am also proficient in using numerical methods to solve mathematical equations that are too complex to solve by hand. All of my research has combined these analytical and numerical skills to investigate mathematical model equations, leading to greater insight about these localized states.

Plans for Evaluation and Dissemination

The project will be evaluated for success by 1) obtaining publishable results 2) relating the results to experimental systems, and 3) having a functional workstation for future projects, which may utilize students.

It is anticipated that the results of this investigation will lead to publication in a refereed physics or mathematics journal. I plan on presenting the results at the Annual Meeting of the Society for Industrial and Applied Mathematics (SIAM) in July 2004. Given opportunity and timeliness, this work is also suitable for presentation at national meetings organized by various other professional societies including the American Physical Society (APS), the American Mathematical Society (AMS), and, to a lesser extent, the Mathematical Association of America (MAA). I will also present the results to the College community at the Mathematics and Physics Seminar. In addition, the results will be communicated with experimentalists and compared with their current observations. The results should also aid the experimentalists in designing additional experiments to test the conclusions.

III. Timeline

Summer 2003

The initial part of the summer will be spent refining the analytical investigation and preparing for the computational investigation. The computations will require modifications of my existing FORTRAN code to solve the equations of interest and any debugging needed to be compatible with new computers and/or computer platforms. The remainder of the summer will be used to perform numerical computations to verify results from the analytical study. I will not be teaching this summer, nor will I be directly involved in the departmental efforts to develop a PDA-enhanced Calculus Course (see Section V Grants).

Academic Year 2003-2004

Upon completion of sufficient corroborating numerical evidence, the results will be written up and submitted to an appropriate journal.

IV. Budget

Linux Workstation and Software	$2000
Faculty Salary	$1500
Total	**$3500**

The bulk of the budget will be spent on acquiring a computer workstation running the Linux operating system. Having a separate workstation has several advantages over running the programs on my personal computer.

First, the codes have been running on Linux/Unix operating systems, which may lead to incompatibility issues if trying to run them on MS Windows. In particular, a large portion of the code is dedicated to graphical output that is currently only compatible with the X Window System found on Linux/Unix platforms. To make the graphical portion of the code compatible would require additional software and a large amount of time to rewrite and debug the code.

My first attempt to have both Linux and Windows on my office computer led to computer problems I experienced last year, namely the hard drive was erased and neither operating system would boot. Although having dual operating systems on one computer is still an option, it poses a particular problem. The software used in classes and in preparation for classes is Windows-based and using my computer to do research calculations in Linux (which may take hours/days), makes all of the Windows programs inaccessible.

Finally, I can create accounts on the Linux workstation for any students working with me on my research. That way they can telnet into the workstation from any computer on campus or from their home. They will have access to the workstation programs and be able to run them directly on the workstation from a remote location. Having a workstation separate from my desktop computer is especially advantageous for student work. Also, for security reasons, a separate workstation would allow me to give students access to a computer that does not contain confidential student and class records.

V. Current and Previous Grants

SBC Ameritech (2002-2004), $18,850
Development of Personal Digital Assistants Enhanced Calculus
This is a departmental grant and I will *not* be working directly on the development of this course nor receiving a faculty stipend.

NCUR/Lancy Grant (2002-2003), $22,500 [faculty stipend, $3500]
Issues related to the expansion of O'Hare International Airport
Proposal written by Jon Johnson and Dianne Chambers

Elmhurst Departmental Initiative Grant (2001), $2500
Development of a Project-Based Technology Course for Preservice Mathematics Teachers

VI. Publications and Presentations

Refereed Journal Publications
C. Crawford, H. Riecke, "Tunable front interaction and localization of periodically forced waves," Phys. Rev. E **65** 066307 (2002).

C. Crawford, H. Riecke, "Oscillon-type structures and their interaction in a Swift-Hohenberg model," Physica D **129** 83 (1999).

Research Presentations
Cooperatively Developing and Teaching a Technology Course for Preservice Teachers, Fourteenth Annual International Conference on Technology in Collegiate Mathematics, November 2001, Baltimore, MD.

Tunable Front Interaction and Localization of Periodically Forced Waves, Society of Industrial and Applied Mathematics Annual Meeting, July 2001, San Diego, CA.

Localization of Periodically Forced Waves, Illinois Section of the Mathematical Association of America Annual Meeting, March 2001, Urbana-Champaign, IL (invited).

Localization of Waves by Temporal Modulation, Annual Meeting of the Division of Fluid Dynamics, American Physical Society, November 1999, New Orleans, LA.

Localized Traveling Waves under the Influence of Temporal Modulation, Fifth SIAM Conference on Applications of Dynamical Systems, May 1999, Snowbird, UT.

Elmhurst College Presentations
Interacting Fronts and Localized Pulses of Periodically Forced Waves, Elmhurst College Department of Mathematics and Physics Seminar, March 2002, Elmhurst, IL.

Patterns and Pulses: Stripes, Squares, and Oscillons in a Model Equation, Elmhurst College Women's Faculty Research Symposium, March 2001, Elmhurst, IL.

Patterns and Pulses: Stripes, Squares, and Oscillons in a Model Equation, Elmhurst College Research Forum, October 2001, Elmhurst, IL.

Poster Presentations
Tunable front interaction in periodically forced waves, Nonlinear Dynamics and Pattern Formation Conference, 2000, Austin, TX.