Elmhurst College

PHY 311-01 Analytical Mechanics

Fall Semester **Earl C. Swallow**

One-dimensional Motion: Interesting Special Cases

For 1-dimensional particle motion, one can develop a general procedure for solving the equation of motion from Newton's Second Law (F = ma) whenever the **net** force depends on **only one** quantity: time (t), or velocity (v), or position (x). The three procedures for the three cases are outlined in this note.

I. F(t)

Think of Newton's Second Law in the form $F(t) = m\frac{dv}{dt}$. Integrating with respect to *t*, the left side yields the impulse integral, and the right side yields the change in momentum (after a change of variable, etc.). Then solve for v(t) and integrate again. This yields a general procedure as follows.

(1) Evaluate the Impulse integral: $I(t) = \int_0^t F(t'')\,dt''$.

(2) $v(t) = v_0 + \frac{1}{m} I(t)$.

(3) $x(t) = x_0 + v_0 t + \frac{1}{m}\int_0^t I(t')\,dt'$

II. F(v)

Think of Newton's Second Law in the form $F(v) = m\frac{dv}{dt}$. Division of both sides by F(v) brings the v's together and makes it possible to integrate with respect to t. The result is a function t(v) which must be inverted and then integrated again. The general procedure then goes as follows.

(1) $t(v) = \int_{v_0}^{v} \frac{m}{F(v'')}\,dv''$

(2) Solve t(v) for v(t).

(3) $x(t) = x_0 + \int_0^t v(t')\,dt'$

III. F(x)

Use the chain rule to write Newton's second Law in the form $F(x) = mv\frac{dv}{dx}$. Integration of this leads to the Work-Energy Principle. Properly defining potential energy then yields the Conservation of Mechanical Energy and the function v(x). This can be re-arranged and integrated to get t(x) which must then be solved for x(t). Finally, one differentiates x(t) to get v(t) if needed. The general procedure then goes as follows.

(1) Evaluate the potential energy function $V(x) = -\int_{x_{ref}}^{x} F(x'')dx''$.

(2) Choose a convenient location for the reference point (x_{ref}).

(3) Calculate the (initial) total Energy (E_0).

(4) $t(x) = \int_{x_0}^{x} \frac{dx'}{\pm\sqrt{\frac{2}{m}[E_0 - V(x)]}}$

(5) Solve t(x) for x(t).

(6) (optional) Differentiate x(t) if you need v(t).