MTH 111-01 Fall 2008 TEST 2A Solutions

1. For $f(x) = -2x^2 + 5x$ and $h \neq 0$, find (10 pts.)

a) Find $f(-1)$ $\quad f(-1) = -2(-1)^2 + 5(-1) = -7$

b) Find $f(x+h)$ $\quad f(x+h) = -2\,(x+h)^2 + 5\,(x+h) = 5h + 5x - 4hx - 2h^2 - 2x^2$

c) Find $f(x+h) - f(x)$

$f(x+h) - f(x) = 5h + 5x - 4hx - 2h^2 - 2x^2 - \left(-2x^2 + 5x\right) = 5h - 4hx - 2h^2$

d) Find $\dfrac{f(x+h) - f(x)}{h}$ and simplify.

$\dfrac{f(x+h) - f(x)}{h} = \frac{5h - 4hx - 2h^2}{h} = 5 - 4x - 2h$

2. A retired couple has \$110,000 to invest and wants to earn \$85,000 per year from the investment. The safer investment earns 6% and the more speculative investment earns 11%. How much should they invest in each of the investments to earn \$8,000 per year. (10 pts.)

	x (9.5% investment)	y (11% investment)	Total
interest	.06x	.11x	8000
Investment	x	y	110000

Then $.06x + .11y = 8000$ and $x + y = 110000$

$$\begin{aligned} 6x + 11y &= 800000 \\ x + y &= 110000 \end{aligned}$$

$$\begin{aligned} 6x + 11y &= 800000 \\ -6x - 6y &= 660000 \end{aligned}$$

$$\begin{aligned} 5y &= 140000 \\ y &= 28000, x = 82000 \end{aligned}$$

3. The demand equation for a certain product is given by $p + 6q = 420$. The supply equation is given by $p = 6q + 60$. Find the market equilibrium price. (10 pts.)

Substitute $p = 6q + 60$ into $p + 6q = 420 => (6q + 60) + 6q = 420 => 12q = 360, q = 30$. The equilibrium price is $p = 6(30) + 60 = 240$

4. Find the equation of the line in slope intercept form: (15 pts)

a) Going through the points $(4, -5)$ and $(-7, -2)$.

$m = \frac{-2-(-5)}{-7-4} = -\frac{3}{11}$. $\quad y = -\frac{3}{11}x + b$ substitute (4,-5): $-5 = -\frac{3}{11}4 + b => b = -5 + 12/11 = -\frac{43}{11}$

$y = -\frac{3}{11}x - \frac{43}{11}$

b) Parallel to $y = -5x + 11$ and going through $(-5, -5)$.

Parallel => equal slopes: $y = -5x + b$, the point is $(-5, -5)$, so $-5 = -5\,(-5) + b => b = -30:$

$y = -5x - 30$

c) Perpendicular to $y = 2x - 50$ and going through the point $(4, -7)$

Perpendicular => neative reciprocal slopes: $y = -\frac{1}{2}x + b$, the point is (4,-7), so $-7 = -\frac{1}{2}4 + b => b = -5$:

$y = -\frac{1}{2}x - 5$

5. Solve algebraically (10 pts.):

$$\begin{aligned} 9x - 2y &= 4 \\ -4x + 13y &= 83 \end{aligned}$$

soln:

$$\begin{aligned} 36x - 8y &= 16 \\ -36x + 117y &= 747 \\ \\ 109y &= 763 \\ y &= 7, \\ 9x - 14 &= 4 \\ x &= 2 \end{aligned}$$

$x = 2, \qquad y = 5$

6. Find the solution algebraically (10 pts.):

$$\begin{aligned} x - 7y + 5z &= -13 \\ -2x + y - 2z &= -5 \\ 2y + 5z &= -8 \end{aligned}$$

soln:

$$\begin{aligned} 2x - 14y + 10z &= -26 \\ -2x + y - 2z &= -5 \\ & yields \\ -13y + 8z &= -31 \quad (1) \\ & Then \\ -13y + 8z &= -31 \\ 2y + 5z &= -8 \\ & yields \\ -26y + 16z &= -62 \\ 26y + 65z &= -104 \quad (2) \\ 81z &= -166 \quad (3) \\ z &= \frac{-166}{81} \quad (4) \end{aligned}$$

Then $2y + 5z = -8 => 2y + 5\left(\frac{-166}{81}\right) = -8$, Solution is: $y = \frac{91}{81}$, Then $x - 7y + 5z = -13 => x - 7\left(\frac{91}{81}\right) + 5\left(\frac{-166}{81}\right) = -13$, so $x = \frac{46}{9}$.

7. A helmet manufacturer has fixed costs of \$6600 per month. Materials and labor for each helmet is \$35, and the company sells the helmet to the dealers for \$60 each. (10 pts.)

a) Write the cost equation.

$C = 35x + 6600$ where x is the number of helmets produced

b) Write the revenue equation.

$R = 60x$

c) Write the profit equation and find the break-even point.

$P = R - C = 60x - (35x + 6600) = 25x - 6600$

The break even point is when the profit P=0: $25x - 6600 = 0 => x = \frac{6600}{25} = 264$ helmets

8. If $f(x) = x^2 - \dfrac{1}{x}$, and $g(x) = 2x + 3$, then find: (10 pts.)

a) $f \circ g(x) = f(2x+3) = (2x+3)^2 - \dfrac{1}{2x+3}$

b) $g \circ g(x) = 2(2x+3) + 3 = 4x + 9$

c) $g \circ f(x) = 2\left(x^2 - \dfrac{1}{x}\right) + 3$

9. Solve for x: (15 pts.)

a) $\frac{3x-1}{4x-9} = \frac{5}{7}$,

Soln: $7(3x-1) = 5(4x-9) => 21x - 7 = 20x - 45 => x = -38$

b) $2x - 3 \le 5x - 7$,

Soln: $2x - 3 \le 5x - 7 => 4 \le 3x => \frac{4}{3} \le x$

c) $\frac{x-1}{2} + 1 > 3x + \frac{1}{3}$,

Soln: $6\left(\frac{x-1}{2} + 1\right) > 6\left(3x + \frac{1}{3}\right) => 3x - 3 + 6 > 18x + 2 => 1 > 15x => \frac{1}{15} > x$