Test 1 MTH 151 Fall 2008 Solutions

I. Find the derivative dy/dx: (5 pts. each)

1. $y = 5x^3 - 7x^2 - (4/x^2) => y = 5x^3 - 7x^2 - 4x^{-2}$
$\frac{dy}{dx} = 15x^2 - 14x + 8x^{-3}$

2. $y = (x^2-1)^4(3x-5)^7$
$\frac{dy}{dx} = (x^2-1)^4 7(3x-5)^6\,(3) + (3x-5)^7 4(x^2-1)^3\,(2x)$

3. $y = \dfrac{x+2}{x-8}$ (simplify)
$\frac{dy}{dx} = \frac{(x-8)(1)-(x+2)(1)}{(x-8)^2} = -\frac{10}{(x-8)^2}$

4. $y = x^2\sec(2x)$
$\frac{dy}{dx} = x^2\sec(2x)\tan(2x)(2) + (2x)\sec(2x)$

5. $y = \cos(\sin(3x^2+x))$
$\frac{dy}{dx} = -\sin(\sin(3x^2+x)) \cdot \cos(3x^2+x) \cdot (6x+1)$

6. $y = [(x^2-1)(x^3-2x)]^6$
$\frac{dy}{dx} = 6[(x^2-1)(x^3-2x)]^5\left[(x^2-1)(3x^2-2) + (x^3-2x)(2x)\right]$

7. $y = \sqrt{x + \sqrt{x^2-5x+2}} => y = \left(x + \left(x^2-5x+2\right)^{1/2}\right)^{1/2}$
$\frac{dy}{dx} = \frac{1}{2}\left(x + \left(x^2-5x+2\right)^{1/2}\right)^{-1/2}\left(1 + \frac{1}{2}\left(x^2-5x+2\right)^{-1/2}(2x-5)\right)$

8. $\cos(x^2+y) = y^2 + xy$
$-\sin(x^2+y)(2x + \frac{dy}{dx}) = 2y\frac{dy}{dx} + x\frac{dy}{dx} + y$
$-\sin(x^2+y)(2x) - \sin(x^2+y)\frac{dy}{dx} = 2y\frac{dy}{dx} + x\frac{dy}{dx} + y$
$-\sin(x^2+y)(2x) - y = \sin(x^2+y)\frac{dy}{dx} + 2y\frac{dy}{dx} + x\frac{dy}{dx} = \left(\sin(x^2+y) + 2y + x\right)\frac{dy}{dx}$
$\frac{dy}{dx} = \frac{-\sin(x^2+y)(2x)-y}{\sin(x^2+y)+2y+x}$

II. (7 pts. each)

1. Find the equation of the line tangent to $y = x^5 - 3x^3 - 4x + 10$ at $x = 2$.
$\frac{dy}{dx} = 5x^4 - 9x^2 - 4$ so $m = 5\cdot 2^4 - 9\cdot 2^2 - 4 = 40$. When $x = 2, y = 2^5 - 3\cdot 2^3 - 4\cdot 2 + 10 = 10$
The equation is $y = 40x + b => 10 = 40(2) + b => b = -70$:
$y = 40x - 70$

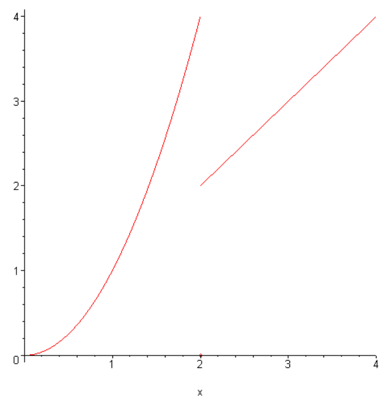
2. Find the equation of the line tangent to $yx + y^2 = 6x$ at $x = 1, y = 2$.
Implicit differentiation: $y + x\frac{dy}{dx} + 2y\frac{dy}{dx} = 6 => (x+2y)\frac{dy}{dx} = 6 - y$
$\frac{dy}{dx} = \frac{6-y}{x+2y}$ At (1,2), the slope is $m = \frac{6-2}{1+2\cdot 2} = \frac{4}{5}$
$y = \frac{4}{5}x + b => 2 = \frac{4}{5} + b => b = 6/5 : y = \frac{4}{5}x + \frac{6}{5}$

IV. Find the limits: (5 pts each)

1. $\lim_{x\to 2}\dfrac{x^2-8x+12}{x^2-4} = \lim_{x\to 2}\dfrac{(x-2)(x-6)}{(x-2)(x+2)} = \lim_{x\to 2}\dfrac{(x-6)}{(x+2)} = \frac{-4}{4} = -1$

2. $\lim_{x\to 1}\dfrac{x^2-5x-6}{x^2-1}$ $\quad UNDEFINED$

3. $\lim_{x\to 0}\dfrac{\sqrt{1+x}-1}{x} = \lim_{x\to 0}\dfrac{\sqrt{1+x}-1}{x}\frac{\sqrt{1+x}+1}{\sqrt{1+x}+1} = \lim_{x\to 0}\dfrac{1+x-1}{x\sqrt{1+x}+1}$



$= \lim_{x\to 0} \dfrac{x}{x\sqrt{1+x}+1} = \lim_{x\to 0} \dfrac{1}{\sqrt{1+x}+1} = \dfrac{1}{2}$ (or recognize as a derivative)

4. $\lim_{x\to -2} \dfrac{x^2+9}{x+3} = \dfrac{4+9}{-2+3} = \frac{13}{1} = 13$

5. $\lim_{x\to 0} \dfrac{x\sin(4x)}{\sin(2x^2)} = \lim_{x\to 0} \dfrac{\sin(4x)}{4x} \cdot \dfrac{2x^2}{\sin(2x^2)} \cdot \dfrac{x(4x)}{(2x^2)} = 1 \cdot 1 \cdot \frac{4}{2} = 2$

6. $\frac{dy}{dx}$ using the limit definition for $y = x^2 - 3x - 7$

$\frac{dy}{dx} = \lim_{h\to 0} \frac{(x+h)^2 - 3(x+h) - 7 - \left(x^2 - 3x - 7\right)}{h} = \lim_{h\to 0} \frac{x^2+2xh+h^2-3x-3h-7-x^2+3x+7}{h} = \lim_{h\to 0} \frac{2xh+h^2-3h}{h}$

$= \lim_{h\to 0} 2x + h - 3 = 2x - 3$

7. $\lim_{x\to 0} \dfrac{\tan x - x}{\sin(2x)} = \lim_{x\to 0} \dfrac{\frac{\sin x}{\cos x} - x}{\sin(2x)} = \lim_{x\to 0} \dfrac{\sin x - x\cos x}{\cos x \sin(2x)} = \lim_{x\to 0} \dfrac{\sin x - x}{\sin(2x)}$

$= \lim_{x\to 0} \dfrac{\sin x}{\sin(2x)} - \lim_{x\to 0} \dfrac{x}{\sin(2x)} = \frac{1}{2} - \frac{1}{2} = 0$

V.. Find the designated limits for f if f is given in the graph below. (6 pts)

a) $\lim_{x\to 2+} f(x) = 2$

b) $\lim_{x\to 2-} f(x) = 4$

c) $\lim_{x\to 2} f(x)$ Undefined/ Does not exist

VI. Find the second derivative (5 pts. each)

1. $f(x) = x^2 - 2x - \frac{1}{x^3} = x^2 - 2x - x^{-3}$

$f'(x) = 2x - 2 + 3x^{-4}$

$f''(x) = 2 - 12x^{-5}$

2. $f(x) = \sin(x^2 - 2x)$

$f'(x) = \cos(x^2 - 2x)(2x - 2)$

$f''(x) = \cos(x^2 - 2x)(2) - \sin(x^2 - 2x)(2x - 2)^2$